MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XLVII. APRIL 6, 1887 No. 6

EDWIN DUNKIN, F.R.S., Vice-President, in the Chair.

The Rev. John Bone, St. Thomas's Vicarage, Lancaster;
W. F. Caborne, Lieut. R.N.R., F.R.G.S., F.R.Met.Soc., 4 Cambridge Gardens, Notting Hill, and Royal Navy Club, Grafton Street, W.;
Campbell M. W. Hepworth, Lieut. R.N.R., 60 Matlock Lane, Ealing Dean; and
William Frederick Smith, F.R.Hist.Soc., 51 Perham Road, West Kensington, S.W.,

were balloted for and duly elected Fellows of the Society.

On the Choice of Instruments for Stellar Photography.

By Howard Grubb, F.R.S.

In view of the Astronomical Congress about to be held in Paris, the subject matter of its deliberations, viz. stellar photography, is naturally attracting much attention at present among astronomers; and I venture to hope that a few notes which I have put together on this subject may not be without interest to the Fellows of the Society. At the same time I desire to premise that I submit them, not with an idea that they possess any peculiar originality, but more as suggestions, and as tending to show the great number and diversity of points which will require to be thoroughly discussed before coming to a satisfactory conclusion as to the best instrumental equipment to satisfy all the conditions which arise in undertaking this new class of astronomical work.

The first and most important question to be decided is whether a refracting or reflecting telescope is likely to give the best results. Theoretically considered, the reflector would appear to have a considerable advantage, for in it there is no *dispersing* of the rays. If the spherical aberration be properly corrected, *all the rays* are brought to the same focus; whereas in the refractor it is not so. If the curve formed by the foci of the different rays of the spectrum be examined, in the case of a refractor, it will be seen that with any possible combination of existing glasses, or arrangements of same, a very small portion only of the whole light can be brought to one focus on the photographic plate; again, the field of the reflector is theoretically flatter; but at the same time, as pointed out by Lieut.-Gen. Tennant at last meeting, a considerable amount of "coma" is to be found in lateral pencils of the reflector, which coma causes distortion of the star discs.

Generally, however, the advantage appears to be on the side of the reflector. It was for these reasons that, when consulted some time ago by Mr. Roberts of Liverpool, I advised his adoption of the reflector in preference to the refractor for photographic purposes.

The value of Lieut.-Gen. Tennant's papers would have been largely increased had he gone a little further, and shown what the *comparative* results would be for the lateral pencils of a refractor. Nor would this be at all impracticable; for, although the result may vary with different constructions of objectives, it would still have been interesting to study the effect on the one ordinary form of double objective (corrected, of course, for chemical rays) such as I have constructed for Dr. Gill, and such as is, I believe, also used by the MM. Henry. At the time I came to the above conclusion respecting the superiority of the reflector I had nothing to guide me but deductions from theoretical considerations. Since then, however, the work of the MM. Henry at Paris, and Dr. Gill at Cape-Town, with refractors, and Mr. Roberts at Liverpool, with a reflector, present some practical results from which to draw further deductions, and while I readily admit that the refractors have done better work than I had expected, I am by no means prepared to admit without further demonstration that under equal conditions they may be expected to do *better* work than the reflectors.

I have not myself had an opportunity of comparing the negatives taken by the reflector and refractor; but I understand from those who have that while the star discs in Roberts' (reflector) photographs show in all parts a slight want of symmetry, there is no decided elongation of the discs up to 80′ from centre of field, whilst in the refractor photographs there is a decided distortion at that distance. The want of perfect symmetry in Roberts' photographs is probably due to the fact that the power of his finder, as compared to the main telescope, is not nearly so great as is used in the case of the refractor.

No doubt Mr. Roberts has not produced the same number of equally fine plates as the MM. Henry; but when we wish to compare results with a view of drawing a comparison between the value of the instruments which have produced those results, it is right and just to take the best specimens of each (not the average), for the inferiority of any examples which are of an excellence less than the best must be attributed to some unfavourable conditions outside the instrument, not in it. An inferior instrument cannot, under any circumstances, give a good result; and Mr. Roberts' best specimens appear to me to compare very favourably with those done by the refractors.

In making the comparison due regard must be had to the fact that the atmosphere of Liverpool is of very inferior quality as compared to that of either Paris or Cape-Town, and also that Mr. Roberts' photographs show an enormously increased number of stars on account of the large aperture of his instrument; but that very large aperture requires of course a still greater degree of perfection of atmosphere in order that its full power may be utilised.*

I do not for a moment dispute the fact that most excellent results can be, and have been, produced by a refractor; but it appears to me that it would be very unwise to spend large sums of money in the installation of several large refractors over the world without some further and more crucial comparative experiments as to the relative fitness of refractor and reflector for this particular work. Mr. Roberts has certainly shown that most excellent work can be done with the reflector; but his instrument is not comparable with the refractors which have been at the same work either as regards size, position, or many other respects.

It is, no doubt, highly desirable that the important work of mapping the heavens should be commenced with as little delay as possible; but it would be equally regrettable if in the beginning a large amount were expended on instruments which proved after a little to be not *the very best* for the purpose. I would venture to suggest, therefore, that a really fair competitive trial be made between refractors and reflectors, to test their fitness for celestial photography, before any definite decision be arrived at. This need by no means impede or hamper the work of the Congress in deciding other questions which could be made dependent on the results of these trials, and six months would probably decide the question.

In the photographing of nebulæ there can be no doubt whatever as to the superiority of the reflector, for in this case increase of angular aperture means increase of light and reduction of ex-

* Since writing above I have received information, founded on practical results, which materially strengthens my views. Professor Pritchard has succeeded in producing photographs with the De la Rue reflector, in which the images at 80′ from centre are sensibly circular, while at same distance in MM. Henry's (refractor) plates the small star discs are ellipses, the proportion of whose major and minor axes are 9 to 4.

posure, and the angular aperture of reflectors can be increased far beyond what is possible in case of refractors.

In connection with the optical portion of the instrument there are a great number of important points which it would be most desirable to discuss; and I mention some of these, not so much with a view of giving my own individual opinion about them, as in the hope of eliciting opinions from others, and having them fairly and fully discussed.

Effect of Increase of Actual Aperture.—Theoretically, so far as star discs are concerned, the brilliancy should increase as the square of the aperture. Long since, in discussing this with Dr. Huggins and Mr. Roberts (before I had any practical results to guide me), I stated that I thought it very likely that the increase of brilliancy would not be in such a high ratio; for practically the discs (of large stars at least) are not mathematical points. Practice, I think, has corroborated this view; but more details are required to make it quite clear.

Effect of Increase of Angular Aperture.—The actual aperture remaining the same, reduction of focus or increase of angular aperture ought *not* to increase the brilliancy of star discs; but, for the same reason as above, it would appear that it does so to some considerable extent.

This, however, is also a question that requires more experiments to enable us to come to a decision upon.

Field of View.—In a refracting telescope the field of view is limited simply by the amount that can be covered by the objective with sufficiently good definition.

In a reflector the conditions are much more complicated.

If the focus be made long the field is flatter and can be used larger; but, on the other hand, the photographic plate and carrier begin to stop out an inconveniently large portion from the centre of the pencil of light.

If the focus be shortened a smaller plate will take in the same angular quantity of field, but that field will not be so flat.

There appears, therefore, to be some proportion that will be found to be best.

So far as present results go, they tend to show that with either a refractor or reflector a field of about $2^\circ \times 2^\circ$ is the maximum attainable with really good definition. Much larger fields may be obtained with combination of lenses; but these are useless for the purpose, as, the pencils not being *all* centrical, a certain amount of distortion is inevitable.

Silver-on-Glass or Metallic Mirrors to be used.—Silver-on-glass mirrors are now so universally used, and so convenient on account of their lightness, that it may seem almost useless to discuss this. However, if the very best result is aimed at, I am inclined to think it might be well to institute some experiments to test the relative reflective powers of speculum-metal and silver for those rays which are used for forming the image on the photographic plate. There are good grounds for supposing that

speculum-metal is more highly reflective for chemical rays than silver.

Long since I obtained photographs of the Moon with the Melbourne telescope, with exposure of from quarter to half a second, using the old collodion process and no peculiarly rapid process. Plates are now made 50 to 100 times as rapid as this old process, but I have never as yet heard of any photographs of the Moon having been taken with a silver-on-glass mirror in $\frac{1}{200}$th or $\frac{1}{400}$th of a second. An idea has been thrown out that the field of a reflector could be improved by modifying the figure of the mirror. This is quite impracticable, as any modification whatever would affect the definition of the central pencil.

It has been objected to reflectors that the film or surface is liable to tarnish, and therefore these instruments are not so stable, and are therefore less suited for long-continued observations.

Metallic mirrors, if the metal is good, I believe to be almost as stable as an object-glass; but even if not so, this is a point of little consequence, for Mr. Roberts tells me that under apparently equal conditions of atmosphere he sometimes obtains three times the number of stars in the same locality as at other times.

With such an enormously variable factor as this to deal with, a slight variation in the condition of the film is of little account.

Best Size of Instrument to adopt.—The larger the instrument the shorter the exposure necessary to obtain an image of a star of any given magnitude; but the difference in this respect will probably not (as I have before said) be on as high a ratio as the theoretical—viz. square of aperture.

On the other hand, the larger the instrument the more difficult will it be to find a night sufficiently good to use it advantageously.

The question of size is therefore mixed up with many others, such as geographical position, accuracy of clockwork, minimum magnitude of star desired to be mapped, rapidity of plates, &c.

In order to determine the best size of instrument it would be necessary to determine first the minimum magnitude of star it is desired to photograph; secondly, for what time the clockwork could be depended upon to go sufficiently accurately to preserve the symmetry of the image.

The problem would then resolve itself into this: What is the *least* size of telescope which would photograph that particular magnitude of stars in that particular period with the quickest plates obtainable?

General Form of the Equatorial Mounting.—If a reflector be used, and more especially if the mirror be metallic, the ordinary so-called German form of equatorial, slightly modified so as to give circumpolar motion, will fulfil all necessary conditions; but in case of a refractor it has been suggested that the old English split polar axis might be more convenient.

In ordinary eye and micrometrical observations the necessity which exists in all equatorials of the German form of reversal is not any great inconvenience, but in stellar photography absolute circumpolar motion is almost a necessity.

It would not be possible to stop during the exposure of a plate, reverse the telescope, and commence again at the other side. It is possible to make the German form circumpolar in this latitude up to the zenith; but to do so for refractors beyond that involves a construction somewhat of the form of Colonel Tomline's, which is clearly impracticable for instruments which have to be transported.

With a reflector, however, the whole telescope being short, the lower end being short in case of a metallic mirror, and the person of the observer being at the upper end, it is perfectly practicable to make the German form circumpolar—at least for these, and even a lower latitude.

Effect of Refraction.—It is very important to study the effect of refraction in disturbing the position of the images; for if, as has been stated, the effect is so great that even in favourable altitudes and with a perfect clock a hand correction must be made every two or three minutes, it would appear almost useless to go to very much trouble about the clock, at least in controlling it so as to keep its rate correct for long periods.

Dr. Dreyer of Armagh has kindly made some calculations for me respecting the effect of refraction. I have only just received his calculations as I send this paper to the printers; but the results are so interesting that they appear worthy of further investigation.

Taking a star near the equator and on the meridian, the difference in Declination for short periods is very small. I have assumed that a shift of star image in plate of $0''{\cdot}5$ would be the least that could sensibly affect the symmetry of image. That $0''{\cdot}5$ error would not occur in Declination for nearly half an hour; but in Right Ascension the error would amount to this quantity in two minutes. The curious part, however, is that this difference in Right Ascension is nearly constant for a considerable distance from meridian: thus, one hour off meridian it is still about $0''{\cdot}5$ in two minutes, and nearly the same at two hours off meridian.

The practical deductions to be drawn from this are:—

(1) That a really perfect controlled clock can be fully utilised, even for long periods, and that it is highly desirable to have such;

(2) That this clock should be provided with an arrangement for introducing very slight changes in its rate.

Clockwork.—This is of course the most important of all the details of an equatorial intended to be devoted to stellar photography.

There are many uniform motion clocks which go smoothly and uniformly for short periods; but for long periods it is abso-

FIG. 1.

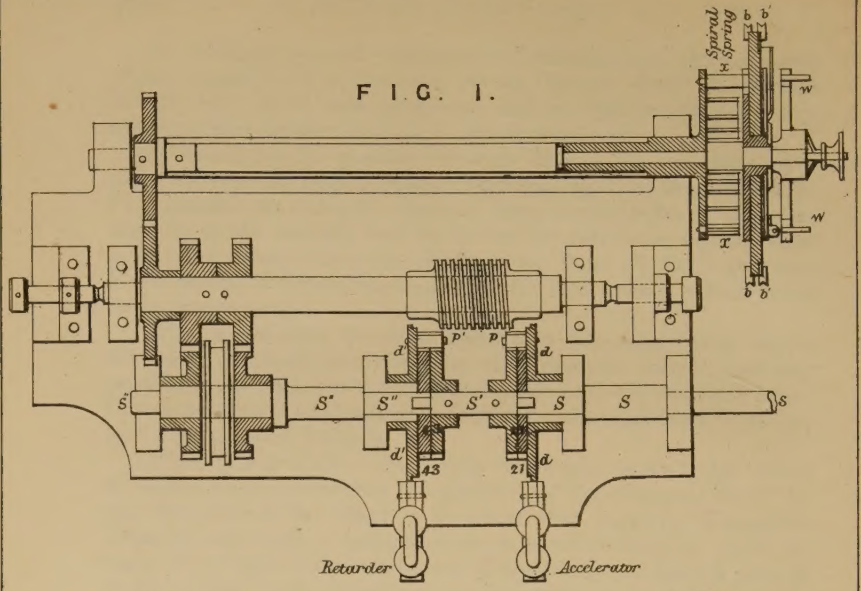


FIG. 2.

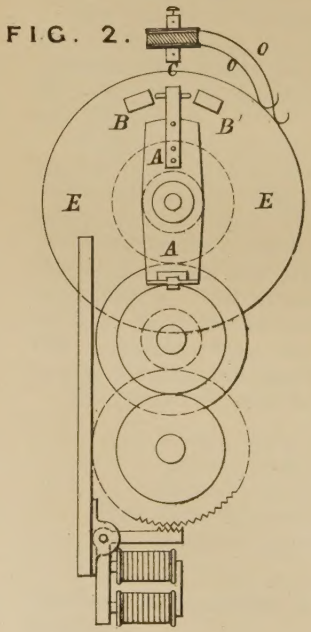
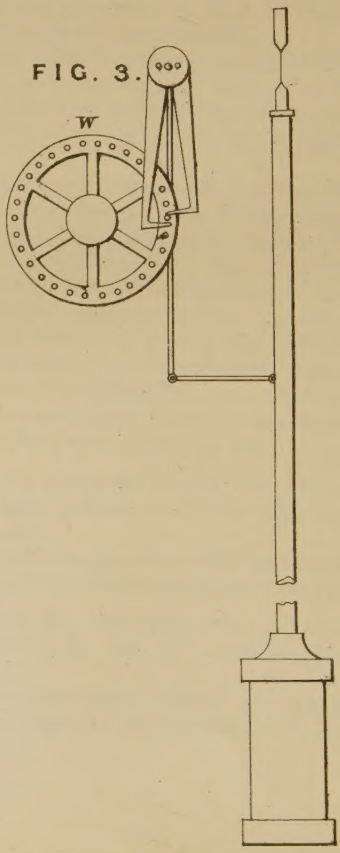


FIG. 3.



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lutely necessary to have some controlling action from a pendulum. The form of control which I have found best consists of two parts—

(1) A detector apparatus for detecting the errors of equatorial clock, using as a standard a mercury or other good pendulum driven by a *remontoire*, so as to be uninfluenced by the incorrect rate of the equatorial clock.

(2) A correcting apparatus for correcting any errors found in the equatorial clock by the detector. This corrector itself naturally consists of two parts—an "accelerator" and a "retarder."

It will be necessary to describe first this accelerator and retarder.

Any of the clock-shafts between the clock and the endless screw which drives the Right Ascension sector is cut into three pieces—Plate 4, fig. 1—S, S′, and S″.

On the end of the shaft S is fixed the wheel 1. On the end of the shaft S′, next S, is fixed the wheel 2, close up to and almost touching 1.

On the other end of shaft S′ is fixed the wheel 3, and on nearest end of shaft S″ the wheel 4. Also, on the shafts S and S″ are placed discs of brass, *d* and *d′*, close to the pair of wheels 1-2 and 3-4; but these discs are *free to turn on shafts*.

On a stud on these discs are pinions, *p* and *p′*, which gear across both wheels of each pair, *p* gearing across Nos. 1 and 2, *p′* across Nos. 3 and 4. In general the whole of this apparatus revolves together and as one piece; but it is possible during the action to arrest the motion of one or other of discs *d* and *d′*, which causes the pinion to revolve on its own axis, and the motion is transmitted to the adjacent wheel through the pinion. If the two adjacent wheels have exactly the same number of teeth the speed remains the same; but if one wheel has (say) one tooth more than the other, a slight difference in speed takes place between the driving part and the driven part of the spindle. For example:

Suppose wheel 1 to have 30 teeth and wheel 2 to have 29 teeth, and that spindle S be revolving at the rate of once in 60 seconds. If now the motion of disc *d* be stopped, the wheel No. 2 and its spindle S′ will be revolving once in 58 seconds instead of once in 60 seconds. By reversing the position of the wheels at the other end of spindle S′ a retardation occurs on stopping its disc *d′*.

If, therefore, the normal rate of spindle S, S′, and S″ be once in 60 seconds, a stopping of disc *d* will cause screw to revolve (as long as disc *d* is held) once in 58 seconds, while a stopping of disc *d′* will cause same to revolve once in 62 seconds.

This stopping is actually accomplished by cutting the edge into a set of very fine teeth (see fig. 2) into which a comb or

the armature of an electro-magnet engages. When therefore a current is sent through the magnet of the accelerator the screw is accelerated $\frac{1}{30}$th part, and when sent through the magnet of retarder the screw is retarded $\frac{1}{30}$th part.

To understand how these currents are sent from the detector, see figs. 2 and 3.

A scape-wheel, W, is mounted on a 60-second spindle of clock, and driven from that spindle by a spiral spring *x x*, fig. 1, so that no error of equatorial clock can affect its rate or the rate of the pendulum. On same clock-spindle behind scape-wheel is mounted an ebonite disc, E E, fig. 2, on which are fixed two brass insulated rings, *b b*, fig. 1, connected metallically with two platinum plates B B′, inserted in face of ebonite disc E E.

Between scape-wheel and ebonite disc is an arm, A A, also mounted on spindle and loose on it, having at its end a little platinum bridge C, of such a length as to fit between the two platinum plates B and B′.

The ebonite disc moves uniformly with the equatorial clock.

The scape-wheel works absolutely correctly with the pendulum and is uninfluenced by error in clock.

The arm A A moves by friction from the ebonite disc, and remains with its bridge exactly between the two platinum discs B and B′ so long as the rate of clock is exactly coincident with the rate of scape-wheel; but if the clock goes the least faster or slower, a pin on the scape-wheel touches arm A, moves bridge C into contact with B or B′, and sends a current through the accelerator or retarder until that error be corrected.

A piece of rock-crystal is inserted between the two pieces of platinum for the bridge to slide on.

The current is guided to brass rings *b b′* by the fine platinum wires *o o′*, which wipe against them.

The time which the accelerator or retarder is kept in action depends on the amount of error it is necessary to correct.

Assuming the proportions given above for teeth of wheels, and that an error—of, say, $\frac{1}{5}$th of a second—is by some means introduced, the corrector will be held in contact for 6 seconds, because for every revolution it is held in contact (that is, for every 60 seconds) a correction of 2 seconds will be made, so 6 seconds' contact will correct $\frac{1}{5}$th of a second.

The advantages of this control are—

(1) It not only brings the rate of the clock right after any temporary disturbance, but it *corrects* any errors that may have already occurred. This is absolutely necessary for photographic work, as otherwise the star would occupy a new position on the plate though rate of clock were corrected. [N.B.—all ordinary forms of fan, frictional governor, isochronous pendulum, &c., only tend to correct the *rate* of the clock, but do not deal with any error already existing.]

(2) The detector arrangement, being attached close to screw-spindle, takes cognisance of all the errors of the various spindles and wheels between clock and screw as well as clock itself.

(3) It is more smooth and equable in its action than any other form, for instead of endeavouring to alter the rate of a heavy moving governor *it allows it to go wrong*; but introduces a corrective arrangement into the gearing so as to *compel the screw* to revolve exactly correctly.

The practical result of this control-clock I find to be that it is admirable for long periods; but the delicacy of the detector part is not as great as I could desire.

Owing to the fact that the linear measure of a second of time on the detector disc (revolving once in 60 seconds) is a small quantity, I find that practically the detector will not detect errors less than $\frac{1}{5}$th to $\frac{1}{10}$th of a second; but it will keep the clock right within *that* error for any number of hours. In other words, the correcting portion of the apparatus (the accelerator and retarder) work perfectly, but the detector is not quite delicate enough. I think it likely that a combination of Dr. Gill's detector (that used at Dun Echt) and my corrector would give the most perfect clock-control possible.

I prefer my corrector because it takes into its influence all the errors of the various connecting shafts, &c., between clock and sector, which Dr. Gill's does not.

Dr. Gill has suggested, in order to avoid all unnecessary wheels and gearing between governors and screw, that the governor spindle should have an endless screw cut on it, gearing *directly* into a toothed wheel on screw-spindle, and that this governor should be driven by a small electric motor and governed from the control by a variable resistance apparatus. The first part of this I consider a most admirable suggestion; but instead of driving by electricity and regulating by the varying resistance, I would still drive by a weight and introduce the correction by my accelerator and retarder, which I find to work so well.

There are many reasons for which I prefer this to the electric driving, among which are:—

(*a*) That whenever possible I prefer gravity (which never fails us) to batteries, as a motive power.

(*b*) That when we apply a corrector which necessitates the alteration of speed of a heavy and quick-moving mass (such as the governor) we are apt to set up a slight swing in the motion. The accelerator and retarder above described are free from that defect.

(*c*) If we adopt the electric driver, the control would have to correct all the errors due to want of constancy of

the battery as well as the mechanical errors of the clock, and as the former would probably amount to 20 per cent. of the whole, while the latter need not exceed $\frac{1}{3}$rd per cent., it is evident that $\frac{5}{6}\frac{9}{0}$ths of the work of the control would be used in correcting a factor (the driving power) which, by adopting gravity, we render practically constant.

The form, therefore, of clockwork I would recommend as probably the best at present possible would be one driven by a weight, the governor gearing directly into a wheel or screw shaft. A *detector* similar to Dr. Gill's at Dun Echt, but driven, not from clock, but directly from screw-spindle, with only one pair of wheels intervening, and a pair of *correctors* on the principle of my accelerator and retarder.

Slow Motion of Equatorial.—For stellar photography it is absolutely necessary to have a slow motion of far greater delicacy and precision than that which suffices for micrometrical work. If in micrometrical work a slight turn of the handle to bring the star central causes the telescope to oscillate very slightly it is of no consequence, as the eye can wait till the oscillation subsides (I am speaking, of course, of extremely small quantities); but in an instrument for stellar photography this is not admissible, as it would cause the star disc to be elongated. Special arrangements, therefore, must be made for the slow motion in the photographic telescope.

Slow Motion in Right Ascension.—For this purpose (beside the ordinary slow motion used to bring star to centre of field) I find the best arrangement is to have another accelerator and retarder, similar to those described above for clock control, and to have the electro-magnets of these connected to a little commutator with two keys held in the hand of the observer. By pressing one the clock-spindle is caused to revolve $\frac{1}{3}\frac{1}{0}$th part faster, and continues at this faster speed so long as the key is held down. By pressing the other a corresponding retardation occurs.

By using this arrangement there is no possibility of causing any jerk or sudden movement in the instrument. The key is held in the hand and is totally independent of the instrument. Pressing the key simply causes the screw to revolve $\frac{1}{3}\frac{1}{0}$th part faster or slower for the time being.

Slow Motion in Declination.—I would be glad if possible to have an equally perfect arrangement for the Declination slow motion; but I do not see how this can be accomplished; and, fortunately, it is not quite so important as it is not so often in use. I incline to think that the German form of slow motion—i.e. a fine screw acting against a stiff spring—would be the best form, only that the strength of the spring is not constant, and, unless made very stiff, does not work well. I have introduced a modification of this which removes these objections.

A B C D (fig. 4) is a portion of the arm attached to telescope

or cradle, on which is planted the block (*b*) forming the bearing of the screw. The nut (*n*) is in the form of a ball, working in a socket on the extremity of the clamp-arm E F G. A short stiff spring (S) is attached to this clamp-arm, bearing not directly against any part of other arm, but against end of a *second screw of same pitch as the main screw*, the nut of which (*o o*) is toothed on edge and works into a wheel of equal size (*p p*) on main screw. The point of this second screw, therefore, advances as much in one direction as the frame A B C D is carried in the other, according as the milled head is turned; and consequently the point of the screw does not sensibly vary in its position with respect to the clamp-arm E F G. A short stiff spring can therefore be used, and the disadvantage above mentioned disappears.

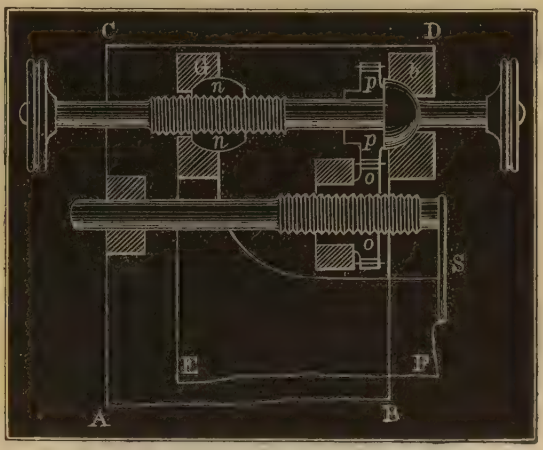

Fig. 4.

The most serious difficulty in using reflectors appears to be that of the want of constancy of the axis of collimation. A mirror cannot be fixed into its cell with the same rigidity as is possible in case of an object-glass, and if the smallest shift occurs during the exposure, any attempt to make the clock drive correctly or to keep a star on cross-line of finder by laborious eye observations are utterly futile.

The shorter the exposure, of course, the less danger of any shift during an exposure; but some system must be devised by which the axis of the finder can be kept in a constant position - not as regards the tube or mechanical fixtures of the reflector, but as respects its true optical axis.

This is not a very simple problem, but I feel quite sure that it is quite capable of being solved.

The necessity of its solution did not arise till now, and therefore no attention has been given to it.

Let it be remembered that we have three possible sources of error—

(1) Lateral shift of mirror, i.e. sliding bodily on its supports.
(2) Flexure of tube.
(3) Tilting of mirror.

By giving the back of the mirror a convex curve equal in radius to concave at front + thickness of disc, the lateral shift has no effect on the collimation. The other two, however, have a very serious effect. An arrangement such as the following would take into account the error in collimation produced by either or both these.

Fig. 5.

Beside, and a little behind, the photographic plate, *but attached* to same supports, let there be a small electric lamp, *e*, enclosed in a little box, and having in front a silvered glass window on which is scratched a minute circle Let a prism and lens, *p*, be mounted

on large mirror itself, in such position as to form an image of that little illuminated circle at *i*. Let the finder telescope be provided near eye-end with a diagonal mirror, *a a*, having a hole in centre, so that the image is thrown out to side of its tube at *i*; and let a lens be mounted in centre hole of that mirror, which will re-form the image of circle at *i'*. On looking into field of finder a ghost image of this circle will be seen in field, and the position of this ghost in field of finder will be affected by flexure of the tube of great telescope or tilting of great mirror. Therefore, if instead of keeping any certain star on fixed cross-lines of finder it be kept in *centre of this ghost ring*, the optical axes of finder and reflector will be kept constant, and the image on photo-plate will be absolutely steady, provided any star in field of finder be kept constantly in centre of that ghost circle.

It is quite possible that a better arrangement than this will be devised; but I mention the first that occurs to me in order to show that it is at least possible.*

Curved Plates.—It appears quite possible that the adoption of slightly curved plates might add considerably to the possible amount of field to be photographed with sufficient sharpness.

The curvature necessary to place the lateral portions of the plate in the place of least confusion would be very slight. In the case, say, of a reflector of 10 feet focus and for a field of 2° the versed sine of this curve would only be about $\frac{1}{60}$th of an inch.

There could be no manipulatory difficulties in coating a plate of this curve, for the common plates in ordinary use by photographers have often more curvature (accidentally) than this.

There would be no danger of any distortion occurring in reducing the observations from these plates if proper precautions were taken, and any extra expense of plates would be more than compensated for if even $\frac{1}{2}$° of additional field could be obtained with the same distinctness.

Many other points beside those above mentioned will have to be considered and discussed before it is possible to come to a definite conclusion as to the best instrumental equipment for stellar photography; but the object of this paper will have been attained if I have succeeded in showing that the subject is one in which new and unexpected conditions are continually presented for our consideration; and, that while we know enough now to be able to say that a certain instrument will give a good result, we really have not as yet sufficient data to enable us to say what will be likely to give the *best* result. For instance, the different conditions, instrumental and otherwise, under which Mr. Roberts and the MM. Henry have been working preclude any idea of drawing a decisive conclusion from a comparison of their results.

It appears to me that the most useful work the Paris Congress

* It may be observed that fig. 5 is only a diagram, and that the comparative size of the various parts of this collimating apparatus, as compared to telescope, are very much exaggerated for clearness' sake

could do would be to formulate a series of experiments which would clear up all debatable points. Arrangements might also be made to enable the work to be carried on immediately after these experiments were completed, the general features of the instrumental equipment to be contingent upon the result of the experiments. If it be possible for me to take advantage of the kind invitation of the French Astronomers, this is the view which I propose to put forward at the Congress.

Remarks on some of the present Aspects of Celestial Photography.
By Professor C. Pritchard, D.D., F.R.S.

When a new science, or a new application of an old one, is in the early stages of its development, as is now the case with stellar photography, any practical remarks connected with its history and progress will certainly possess a peculiar interest when, in the distant future, the art shall have been brought to comparative perfection. It is under this view that I venture to occupy for a few minutes the attention of the Society.

We are greatly indebted to General Tennant for his recent contribution to our theoretical knowledge of the nature and position of the foci of cylindrical pencils of light incident on parabolic mirrors, when the axes of these pencils are inclined to each other at considerable angles. By considerable angles I mean such as extend through one to three, or even four, degrees. These investigations have a very close and important bearing on questions relating to the angular extent of the photographic pictures of the heavens, which may be hoped for through the instrumentality of reflecting telescopes. One result which I gather from General Tennant's labour is that, even at angles slightly exceeding one degree from the axis of the mirror, discernible astigmatism and distortion may be expected to commence; and if such be found to be the actual result in practice, then photographic pictures of the distribution of stars will be necessarily confined within very narrow limits indeed; and if the whole heavens are to be photographically charted, then the number of the plates will become unavoidably enormous, inasmuch as no single plate could be expected to contain more than five or six square degrees, in at all events such perfection as not to offend the eye, and remind it forcibly and continuously of the comparative imperfection of the means adopted for the formation of the pictures. Passing for a moment from these inauspicious expectations of aid from telescopic mirrors, I find from a recent printed note of Mr. Common that the photographic results, at present derived from specially constructed object-glasses, are, in respect of accuracy of form and extent of available field, not much more hopeful than those predicted by mathematical in-

vestigation in the case of reflectors. Mr. Common, speaking of the really beautiful results obtained by MM. Henry, says: "Looking now at a picture of one of the faintest stars on the plate, and about the centre of the field, these images are quite distinct and perfectly symmetrical; it is only when we get about 1° from the centre that distortion begins. . . . We must therefore consider that the field of this instrument is a circle of about 2° diameter . . ."

It was with a view of ascertaining, in respect of circularity of the stellar images and available extent of the field, *within the limits of agreeable representation*, that I recently commenced some experiments with the De la Rue reflecting telescope of 13 inches aperture. The ordinary plate-holder of this instrument is a circle of two inches only in diameter, at a focal distance of 120 inches, and consequently it comprehends a field scarcely exceeding a single degree in diameter. Beautifully perfect as this field is, and presenting images of the fainter stars admitting of refined measurements, suitable even for the purposes of stellar parallax, it affords no adequate extent of picture practically adapted for the larger scheme of charting the heavens. I accordingly caused to be constructed a camera of more than double this diameter, and affording a circular field extending to about 2°·7; and although its construction with home resources was very rough and temporary in its character, nevertheless a series of negatives was obtained exhibiting not the slightest discernible deformity in the figures of the stars. This encourages me to hope that the field may be safely extended yet further with suitable appliances, and possibly sufficiently so for efficient and general charting. One of these negatives thus considerably passing the angular limits indicated by the mathematical analysis, I have very recently submitted to the judgment of some members of the committee delegated by the Society to inquire into such matters.

Independently of this, at a meeting of the Photographic Committee of the Royal Society, comprising several of the members of the former body, it was proposed that experiments should be made with two mirrors of widely different focal lengths in order to determine what is the greatest extent of available field to which photographic pictures can be thereby carried. Mr. De la Rue, with his well-known and thoughtful generosity, undertook to provide two such mirrors for the determination of the question, and he has placed them in my hands at Oxford with the view of mounting them temporarily on the solid frame of his reflector in the University Observatory. The aperture of each is 15 inches, with focal lengths respectively of 80 inches and 120 inches. Cameras are now in process of construction capable of photographing fields of 3° 20′, and extending to nearly 5° in diameter. What the result may be cannot be told until after the inquiry has been completed.

There is also another question of some importance to be

decided. It relates to the conical or trumpet-shaped form of the telescope tube, indicated in General Tennant's paper already referred to. There can be no doubt that owing to the ordinary cylindrical form of the tube very much light from stars whose images are formed near the edge of the field is cut off from incidence on the mirror. This fact will also necessarily interfere with the correct estimation of the magnitude of such stars. On the other hand, it is indicated in the analysis referred to that there is much probability of astigmatism and distortion in the images of these widely angular pencils, and it may even turn out that deformation is partially prevented by the mouth of the ordinary cylindrical tube; for it is one of the results of the mathematical investigation that a diaphragm placed within the tube may conduce in some degree to the flatness of the field. All this is a subject for further practical inquiry, and, as Mr. Common reminds us by a forcible remark, "there is not (at present) much information available, and what there is is such as rather raises than removes doubts."

Should it so happen that considerably wider fields than those indicated by the mathematical analysis can be successfully pictured on the plates, this need not be regarded as contravening the correct interpretation of General Tennant's research. For the photographic images are not in any sense mathematical points; and as to their physical genesis, we are greatly in the dark. One thing is certain, that they grow under continuous exposure; but whether that growth is due to the diffusion of the light, or in some degree to the unsteadiness of the atmosphere or of the instrument, or to some molecular action, extending as it were from a centre of disturbance, who can at present tell?

One conclusion, I think, is clearly deducible from the foregoing remarks, to the effect that the method and final plan of a general charting of the heavens is not yet wholly ripe for decision. For my own part, I do not think it a chimerical hope that before long it may be within the power of a single observatory, provided with two, or it may be three, capable assistants, to procure a series of charts of the heavens, extending from the Pole to the Equator, photographed on an efficient scale, without unpleasant distortion, extending to stars nearly as faint as the fifteenth magnitude, and reproduced in a permanent form nearly to the fourteenth, and then circulated among astronomers within a space of time not exceeding four years. The plates, however, must necessarily be duplicated by a second instrument for the purposes of rectification.

These, however, are questions which await discussion at the approaching Conference at Paris. Meanwhile there are two mottoes applicable to the present condition of the matter before us. The one is "*Forwards!*" and the other is "*Festina Lente.*"

1887, *April* 5.

On the Variations of Level and Azimuth of the Transit Circle at the Royal Observatory, Greenwich. By H. H. Turner, M.A., B.Sc.

In Vol. XXIX. of the *Memoirs* of the Royal Astronomical Society, Mr. William Ellis has given a very complete account of the changes of level and azimuth errors of the Transit Circle during the early years of its history (1851–1858). He found that there were fluctuations in the position of the instrument in both these elements which followed very closely the changes of external temperature, especially those of long period, such as the annual variation. As the changes of position of an instrument which has been in constant use for so long a period are of interest, the following tables have been recently formed, showing the simple arithmetical mean of the adopted level errors in each month, and of the adopted azimuth errors in each month, for the period 1851–1884. From the means for each month that for the year has been formed (in the column on the right); and similarly the means for the separate months have been formed (in the bottom line), excluding those years in which the changes mentioned in the notes have been made.

Mean Values of Adopted Level Error of the Transit Circle for each month and for each year since its erection.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
1851	− 2″64	− 3″38	− 4″01	−4″53	−4″83	−5″91	−4″56	− 4″31	− 2″71	− 1″75	− 0″61	− 1″49	− 3″39
1852	− 2·20	− 2·72	− 2·79	−3·14	−3·99	−4·61	−6·12	− 2·64	− 0·52	− 0·43	− 1·29	− 1·23	− 2·64
1853	− 1·37	− 1·07	− 1·75	−2·56	−3·51	−3·38	−2·92	− 1·60	− 0·30	0·00	+ 0·63	+ 0·86	− 1·41
1854	− 1·38	− 1·44	− 1 72	−2·42	−1·79	−2·50	−2·83	− 1·46	− 0·31	+ 1·75	+ 2·59	+ 2·06	− 0·79
1855	+ 1·43	+ 1·70	+ 0·95	−0·02	−0·72	−0·81	−0·85	+ 0·09	+ 1·94	+ 2·93	+ 3·52	+ 2·47	+ 1·05
1856	+ 2·22	+ 1·78	+ 1·77	+0·12	−0·10	−0·73	−0·70	+ 0·29	+ 2·25	+ 3·13	+ 3·78	+ 3·15	+ 1·41
1857	+ 2·70	+ 2·07	+ 1·61	+0·82	−0·13	−0·92	+0·59	+ 1·77	+ 3·49	+ 4·84	+ 5·59	+ 4·66	+ 2·26
1858	+ 4·18	+ 4·16	+ 2·73	+1·70	+1·50	+0·02	+2·32	+ 3·43	+ 4·29	+ 5·61	+ 5·62	+ 4·96	+ 3·38
1859	+ 5·00	+ 4·20	+ 3·57	+3·49	+3·17	+1·58	+1·89	+ 4·71	+ 6·63	+ 7 39	+ 7·40	+ 6·95	+ 4·66
1860	+ 6·37	+ 5·81	+ 5·08	+4·42	+2·86	+3·44	+3·37	+ 4·22	+ 5·42	+ 5·85	+ 6·79	+ 6·33	+ 5·00
1861	+ 5·87	+ 4·94	+ 4·60	+5·58	+3·71	+3·16	+4·70	+ 5·11	+ 7·03	+ 7·23	+ 8·44	+ 7·63	+ 5·67
1862	+ 6·93	+ 6·48	+ 5·11	+4·79	+3·20	+3·72	+3·99	+ 4·89	+ 6·02	+ 7·16	+ 8·01	+ 7·34	+ 5·64
1863	+ 6·87	+ 7·17	+ 6·38	+5·65	+4·86	+4·87	+5·98	+ 6·18	+ 8·45	+ 8·29	+ 8·84	+ 8·68	+ 6·85
1864	+ 8·29	+ 8·09	+ 7·39	+4·76	+5 66	+6·17	+6·36	+ 7·66	+ 9·08	+ 9·61	+10·05	+10·07	+ 7·77
1865	+ 9·79	+ 8·90	+ 8·61	+6·33	+6·16	+6·67	+7·09	+ 9·26	+ 9·10	+10·97	+11·30	+10·95	+ 8·76
1866	+10·59	+ 9·88	+ 9·72	+8·32	+8·67	+7·35	+8·42	+ 9·25	+10·65	+10·79	+10·92	+10·98	+ 9·63
1867	+10·43	+ 9·63	+ 9·91	+7·90	+6·94	+7·05	+7·84	+ 7·32	+ 9·43	+12·79	+10·72	+10·36	+ 9·19

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	″	″	″	″	″	″	″	″	″	″	′	″	″
1868	+ 9·41	+ 8·57	+ 8·31	+ 7·46	+ 6·26	+ 6·37	+ 7·06	+ 10·15	+ 10·78	+ 12·44	+ 12·61	+ 11·51	+ 9·24
1869	+ 11·57	+ 10·23	+ 10·74	+ 8·02	+ 9·02	+ 8·28	+ 7·31	+ 9·45	+ 9·98	+ 11·89	+ 11·90	+ 11·82	+ 10·02
1870(*a*)	+ 10·91	+ 10·69	+ 9·60	+ 8·49	+ 7·82	+ 7·61	+ 7·94	+ 10·41	+ 10·88	+ 11·39	...	+ 2·77	...
1871	+ 2·30	+ 0·49	+ 0·08	− 0·07	− 0·70	− 0·29	− 1·34	− 1·01	− 1·65	+ 2·95	+ 2·99	+ 2·27	+ 1·11
1872	+ 1·84	+ 0·98	+ 0·88	− 0·49	− 0·68	− 2·31	− 1·67	+ 0·65	+ 1·53	+ 2·88	+ 1·83	+ 2·40	+ 0·65
1873	+ 1·39	+ 1·13	+ 0·09	− 0·17	− 1·20	− 2·32	− 2·45	− 0·63	+ 1·38	+ 2·44	+ 1·82	+ 1·72	+ 0·27
1874	+ 1·15	+ 1·19	− 0·24	− 1·74	− 0·33	− 1·54	− 1·85	− 0·21	+ 0·69	+ 1·73	+ 2·51	+ 2·41	+ 0·31
1875	+ 0·43	+ 0·67	− 0·37	− 0·95	− 2·53	− 2·46	− 1·94	− 2·41	− 0·69	+ 0·70	+ 0·96	+ 0·25	− 0·69
1876	− 0·46	− 1·25	− 0·89	− 1·57	− 2·10	− 3·99	− 3·95	− 2·29	+ 0·08	+ 0·03	+ 0·68	− 0·07	− 1·31
1877(*b*)	− 1·23	− 1·86	− 2·29	− 3·09	− 3·53	− 5·76	− 4·48	− 3·92	− 1·56	− 1·46	− 1·00	(− 1·03)	− 2·60
1878	− 1·85	− 3·08	− 2·40	− 3·68	− 4·45	− 6·29	− 5·47	− 3·84	− 2·27	− 1·82	− 1·18	− 1·70	− 3·17
1879	− 2·41	− 3·92	− 4·16	− 4·64	− 5·05	− 5·47	− 6·62	− 6·61	− 5·05	− 3·95	− 3·07	− 3·12	− 4·51
1880	− 4·62	− 5·52	− 5·85	− 6·33	− 7·46	− 7·98	− 8·42	− 7·76	− 5·76	− 4·08	− 3·76	− 5·04	− 6·05
1881(*c*)	− 4·46	− 5·37	− 6·34	− 7·39	− 8·49	− 8·53	...	− 0·75	+ 0·51	+ 1·81	+ 0·87	+ 0·93	...
1882	+ 0·44	− 0·22	− 2·48	− 2·12	− 3·22	− 3·31	− 2·24	− 1·39	+ 0·02	+ 0·85	+ 0·88	+ 0·23	− 1·03
1883	− 0·59	− 1·37	− 1·18	− 2·84	− 3·20	− 3·61	− 3·49	− 2·77	− 1·26	− 0·13	− 0·06	− 0·05	− 1·71
1884	− 1·34	− 1·50	− 2·92	− 2·46	− 3·72	− 3·96	− 4·42	− 3·97	− 1·93	− 0·66	− 0·61	− 1·22	− 2·39
Mean	+ 2·91	+ 2·34	+ 1·83	+ 0·96	+ 0·40	− 0·12	+ 0·16	+ 1·34	+ 2·87	+ 3·92	+ 4·19	+ 3·75	+ 2·05

Mean Values of Adopted Azimuth Error of the Transit Circle for each month and for each year since its erection.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
1851	−7·13	−8″·29	−7″·28	−6″·72	−6′·12	−3″·19	−1″·61	− 0″·72	+ 0″·02	− 1″·21	−3″·97	−5″·62	−4″·32
1852	−6·54	−7·96	−8·45	−7·82	−7·57	−6·00	−2·77	− 1·32	− 0 36	− 1·32	−2·70	−3·40	−4·68
1853	−4·28	−7 68	−9·09	−9·82	−6 63	−4·79	−2·35	− 0 84	− 0·57	− 0·87	−3·46	−6·05	−4·70
1854	−7·89	−9·34	−7·44	−8·16	−7·04	−5·47	−3·57	− 2·74	− 2·07	− 2·39	−3·02	−5·12	−5·35
1855	−7·70	−9·25	−9·45	−7·69	−7·59	−4·59	−2·59	− 1·03	− 0·68	+ 0·32	−1·67	−5 09	−4·75
1856	−5·76	−7·28	−7·61	−6·53	−6·21	−2·87	−1·40	+ 1·05	+ 0 18	+ 0·49	−3·21	−3 12	−3·52
1857	−5·34	−6·19	−6·61	−5·42	−3·75	−1·96	+0·31	+ 2·43	+ 3·34	+ 2·39	+0·62	−0·61	−1·73
1858	−3·02	−5·55	−7·08	−6·30	−4·46	−0·90	+0·16	+ 1·85	+ 1·96	+ 0 65	−1·35	−1·54	−2·13
1859	−3·37	−3·97	−4·99	−5·36	−4·32	−1·57	+2·47	+ 4·03	+ 3·75	+ 2·73	−0·18	−1·72	−1·04
1860	−2·78	−5·27	−5·60	−7·12	−4·60	−3·55	−2·53	− 0·59	− 1·02	− 1·54	−2 81	−3·95	−3·45
1861	−6·89	−6·71	−7·19	−6·11	−7·31	−4·16	−1·43	+ 0·77	+ 0·65	+ 0·55	−2·11	−3·65	−3·63
1862	−5·31	−4·21	−5·86	−4·80	−3·44	−2·51	−0·68	+ 0·26	+ 0·76	+ 0 01	−2·60	−3·19	−2·63
1863	−5·22	−4·14	−6 01	−5 45	−5·02	−1·93	+0·95	+ 1·52	+ 0 57	+ 0·43	−1·30	−3 26	−2·41
1864	−6·33	−5·75	−7·01	−5·77	−3·30	−2·33	−0·46	+ 1·51	+ 1·85	− 0·28	−2·47	−2·93	−2·77
1865	−5·29	−6·45	−7 67	−4·92	−3·66	−0·85	+1·23	+ 1 73	+ 4·15	+ 0·59	−0·92	−1·42	−1·96
1866	−2·27	−4·27	−5·08	−3·41	−3·34	+0·50	+2·46	+ 2·25	+ 2·66	+ 1·13	−1·19	−0·80	−0·95
1867	−3·94	−1·87	−3·35	−2·49	−1·69	+0·73	+0·45	+ 3·23	+ 4·06	+ 0 72	−1·28	−3·84	−0·77
1868	−5·58	−5·00	−5·19	−4·92	−2·71	+0·32	+3·22	+ 4 21	+ 3·55	+ 3·09	+0·30	+0·54	−0·68
1869	−2·56	−2·10	−4·62	−3·47	−2·08	−0·94	+2·57	+ 3·14	+ 3·35	+ 2·50	−0·67	−2·36	−0·60
1870	−3·97	−5·60	−5·93	−4·87	−2·74	+0·66	+2·59	+ 3·54	+ 2·50	+ 1·89	$\left\{\frac{-0·35}{-1·11}\right\}$ (a)	−3·38	−1·34

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	″	″	″	″	″	″	″	″	″	″	″	″	″
1871	−5·48	−4·18	−4·09	−3·96	−3·25	−1·20	+1·05	+3·41	+4·09	+2·18	−0·94	−2·96	−1·28
1872	−2·30	−2·41	−2·59	−1·90	−1·96	+0·07	+2·77	+4·92	+4·66	+2·34	+1·24	−0·16	+0·39
1873	−0·82	−3·72	−3·11	−1·61	−1·38	+0·10	+3·44	+4·89	+3·84	+2·68	−0·16	−0·89	+0·27
1874	−0·33	−1·72	−1·01	−0·04	+0·77	+3·31	+5·36	+5·36	+5·88	+4·37	+2·52	−0·56	+1·99
1875	−0·65	−1·08	−1·64	−0·31	+2·08	+4·65	+6·42	+8·78	+9·33	+8·09	+5·27	+2·87	+3·65
1876	+1·95	+1·91	+2·18	+2·97	+3·32	+5·68	+9·19	+10·79	+9·45	+9·16	+6·76	+6·66	+5·83
1877	+5·77	+4·93	+3·44	+3·67	+4·05	+7·38	+8·92	+11·19	+10·60	+9·43	+8·45	$\left\{\frac{-1·72}{+5·97}\right\}$ (b)	+6·35
1878	−2·95	−2·61	−1·86	−2·09	−0·23	+0·06	+3·44	+5·30	+4·91	+4·54	+0·49	−1·97	+0·59
1879	−2·93	−2·96	−1·70	−2·59	−1·14	+1·59	+2·54	+4·75	+4·83	+4·38	+1·40	−1·93	+0·52
1880	−3·09	−1·94	−0·31	−0·23	+1·06	+2·13	+4·47	+5·61	+7·27	+4·85	+2·07	+1·62	+1·96
1881	−1·93	−1·40	−0·83	−1·25	+0·63	+2·83	+5·79	(+8·16)(c)	+7·80	+6·11	+5·82	+4·47	(+3·02)
1882	+3·62	+3·35	+2·96	+3·39	+5·30	+6·15	+8·36	+9·59	+9·26	+8·70	+6·44	+4·68	+5·98
1883	+3·47	+2·91	+0·84	+1·63	+4·38	+6·48	+7·06	+9·64	+10·07	+9·11	+6·74	+5·93	+5·69
1884	+5·43	+4·56	+4·61	+4·45	+6·00	+7·86	+8·91	+10·99	+11·23	+10·59	+9·29	+8·17	+7·67
	−3·27	−3·84	−4·20	−3·63	−2·45	−0·30	+1·85	+3·38	+3·58	+2·55	+0·23	−0·96	Mean

Notes.

(*a*) 1870, Nov. 19.—Eastern Y raised and sheet of thin paper placed under it. The Y was adjusted in azimuth by means of the collimators. The two numbers given for azimuth are the means for the two portions of the month.

(*b*) 1877, Dec. 4.—Western Y moved slightly to N to reduce azimuth error. The upper number for azimuth is the actual monthly mean. The lower has been formed by differencing from 1876 and 1878, for use in the annual variation.

(*c*) 1881, July 27–29.—Eastern Y raised and sheet of paper removed. Y adjusted in azimuth by reference to collimators.

The last columns exhibit the secular change in the position of the instrument. The lowest lines exhibit, after correction for that part of the secular change which takes place throughout the year, the annual variation. I have thought it of interest to express this variation in harmonics of annual and semi-annual period, giving at the same time the corresponding expression for the temperature of the air, whose connection with these fluctuations Mr. Ellis has already pointed out. The results are as follows—the unit of time being one year, and the months being assumed of equal length :—

Level $-1''{\cdot}96 \sin (298^\circ\ 45' + t.\ 360^\circ) - 0''{\cdot}45 \sin (67^\circ\ 55' + 2t.\ 360^\circ)$.

Azimuth $+3''{\cdot}75 \sin (204^\circ\ 7 + t.\ 360^\circ) + 0''{\cdot}44 \sin (341^\circ\ 30 + 2t.\ 360^\circ)$.

Temperature $12^\circ{\cdot}22 \sin (247^\circ\ 13 + t.\ 360^\circ) + 1^\circ{\cdot}23 \sin (30^\circ\ 42 + 2t.\ 360^\circ)$.

I have only given two terms of the harmonic series, although the residuals of the means of so many years show traces of being capable of further analysis. They are as follows :—

	Level.	Azimuth.	Temperature.
	$''$	$''$	$^\circ$
January	+0·01	−0·13	0·0
February ...	+0·01	+0·06	+0·3
March	+0·04	−0·07	−0·7
April	−0·13	+0·08	+0·5
May	+0·09	−0·04	−0·2
June	+0·02	+0·05	+0·4
July	−0·04	−0·03	−0·3
August	−0·02	−0·02	−0·2
September ...	+0·03	+0·02	+0·3
October	0·00	+0·15	+0·7
November ...	−0·01	−0·32	−1·3
December ...	−0·05	+0·29	+0·7

If these changes are due to temperature, which we may suppose to be only gradually taken up by the masses of stone and metal, and the ground on which the instrument stands, we should expect the lagging effect behind the temperature which is exhibited by the azimuth. The difference of epoch for the first harmonic is about 43°, or about a month and a half; and for the second 48°.

We may get some idea of the kind of phenomena to be expected by studying the law generally assumed for underground temperature, though we must remember that this law is deduced for a homogeneous infinite body bounded by a plane surface, while we are here dealing with a mass of variable density and shape, and that we do not know more than very

roughly which part of it contributes most to the effect under consideration.

A wave of temperature expressible by the term

$$A \sin (nt + \alpha)$$

reaches the depth x under the form

$$Ae^{-kx\sqrt{n}} \sin (nt - kx\sqrt{n} + \alpha),$$

where k is a constant depending on the conductivity of the soil.

If the variation of azimuth were due to the variation of temperature at a definite depth below the surface of some part of the instrument, we should expect the lagging for the second harmonic to be about $\sqrt{2}$ times that for the first harmonic. We see, however, that, although the former is decidedly the greater, it is not quite so large in proportion as this. Again, the ratio of the coefficient of the second harmonic to that of the first is a little greater than the corresponding ratio for temperature, whereas we should expect it to be a little less.

But it is probable that this case of the temperature at a definite depth is not fairly comparable with that before us; we are more likely to get a fair analogy by considering the summation of a series of small changes at continually varying depths, i.e. by integrating the above expression in some way.

Simple integration with respect to the depth from the surface to a considerable distance within the earth would give

$$\int_0^\infty Ae^{-kx\sqrt{n}} \sin (nt - kx\sqrt{n} + \alpha) dx = \frac{A}{k\sqrt{n}} \sin \left(nt - \frac{\pi}{4} + \alpha\right),$$

which would represent, say, the elevation of the surface owing to expansion below. And in this sort of integrated effect we notice an important difference from the former; viz. the lagging is constant for waves of all period, and is nearly 45°. Without laying too much stress on the point, it is interesting to notice that the actual lagging is 43° and 48° for the two harmonics. The truth thus apparently lies between this hypothesis and the former, as we might expect.

On proceeding to consider the level, however, we find that, so far from its lagging behind the temperature, it actually precedes it by nearly as much as the azimuth falls behind. In considering a periodic term it is of course always possible to express it as either preceding another of like period by, say, an eighth-period, or following it by seven-eighths; or if we are at liberty to consider a crest of one to correspond to either a crest or trough of the other, we may express the same anticipation of one-eighth of a period as a lagging of three-eighths. The only reason for considering a decrease of level to correspond to an increase of temperature is that such is found to be the case in the irregular and sudden changes which occur from day to day;

but it is of course quite possible that these rapid changes may be caused by the expansion of quite a different part of the instrument, e.g. much nearer the surface of some part of it. We have then the alternative of considering that the level lags at least $4\frac{1}{2}$ months behind the temperature, perhaps $10\frac{1}{2}$ months; and when we consider that this would mean that the seat of these changes would be at a depth of about 24 feet (or perhaps 56 feet) if the conductivity of the parts of the instrument be even so small as that of rock, this does not seem very likely.*

I was led therefore to speculate as to whether this apparent anticipation of the temperature might not be real; and I venture to hazard the suggestion that a difference of conductivities in the two piers of the transit circle would produce such an effect. If we differentiate the expression

$$Ae^{-kx\sqrt{n}} \sin (nt - kx\sqrt{n} + \alpha)$$

to k we obtain

$$-Ak\sqrt{n}\, e^{-kx\sqrt{n}} \sin \left(nt - kx\sqrt{n} + \frac{\pi}{4} + \alpha\right);$$

that is, this differential effect anticipates the former by an eighth-period, and the relative value of the terms of short period for which n is larger is increased in the proportion of $\sqrt{n}$ to 1. This result is not to be taken numerically, but only as a very rough qualitative illustration of the way in which the temperature possibly influences the level. It will be noticed that the supposition of a difference of conductivities will thus explain the apparent anticipation by the level of the change of temperature, and at the same time the large relative value of the second harmonic in the case of level, which is much more marked than in the case of azimuth.

Now, we have above differentiated the expression for the temperature at a definite point within the mass; but it is probable that some process of integration should first have been employed as in the case of azimuth. We have however previously studied the general effect of integration, and know that it is in the opposite direction to that just mentioned. Generally we should expect these two causes—the gradual filtering of the temperature into the solid mass producing an effect of retardation, and the difference of conductivity between the various parts producing an anticipatory effect, to counterbalance one another to a considerable extent, both in the case of azimuth and level. They have been assigned respectively to the one and the other, first for simplicity, and secondly because the *amount* of the effect to be expected from these respective causes compared with the actual differences of epoch observed in the two cases seems to point to the fact that the variation of azimuth is mainly due to the first, and of level to the second. If it be worth while to

* See *Greenwich Observations* for 1860. Reduction of the observations of the deep-sunk thermometers, by Prof. Everett.

hazard further conjecture as to the reason of this difference of behaviour of the instrument in the two elements we might suppose more definitely that the level error is caused by a warming of the eastern pier more rapidly than the western, whose conductivity is not so great; so that it expands vertically and affects the level, while at the same time the lateral expansion, being symmetrical about the central vertical plane which passes nearly through the pivots, would not affect the azimuth. The piers, or that part of them at least which supports the pivots, not being very thick, the lagging due to gradual conduction will not be very large, and we have the effect of difference of conductivity nearly free from any counteraction.

On the other hand, we must refer the azimuthal variation to changes of temperature probably at some depth below the surface of the soil, which do not happen to affect the level appreciably.

I have examined the variations of level and azimuth at other observatories for a few years to see if there is any similarity between these fluctuations for different instruments. It is somewhat difficult, without expending more time than I can well spare at present, to disentangle their annual variations from changes, secular and irregular, and generally larger than those noticed at Greenwich. I look forward with interest to a discussion by Mr. Finlay of the errors of the Cape Transit Circle promised in the last volume of "Cape Observations" (1879–81). Roughly speaking, there does not seem to be much similarity between the fluctuations of position of different instruments; for instance, the difference of epoch, which has been chiefly considered above, is very variable. And indeed Mr. Ellis pointed out in his paper above referred to how largely the variations of the present Transit Circle differ from those of Troughton's Transit, which occupied nearly the same site. As I have already said, however, in suggesting the above explanations of the changes in position of the Greenwich Transit Circle, we have at our disposal two causes which, combined in varying relative proportion, are capable of explaining fluctuations following or anticipating the temperature by very different periods of time.

On the Formulæ for Computing the Apparent Positions of a Satellite, and for Correcting the Assumed Elements of its Orbit. By A. Marth.

The methods of computation connected with the investigation of a satellite's orbit are comparatively simple and convenient, if they are duly selected to suit directly the coordinates furnished by the observations. If polar-coordinates have been observed, there exists apparently no good reason for not employing polar-coordinates also in the corresponding computations. The present

paper contains the formulæ, which seem to me the most advantageous and handy for computing the apparent positions and the corrections of the assumed elements in the chief cases which come under consideration.

Referring the plane of the satellite's orbit to the plane parallel to the terrestrial equator, let N denote the longitude or Right Ascension of the ascending node; J the inclination; Q the angle between the line of apsides and line of nodes or the departure of the lower apside from the node; $e = \sin\phi$ the eccentricity: μ, ϵ, v the mean, eccentric, and true anomaly of the satellite; $u = Q + v$ its true longitude in the orbit reckoned from the node; $u_0 = Q + \mu$ its mean longitude; r the radius vector; a the major semi-axis of the orbit or the satellite's linear mean distance from the planet's centre; $\frac{r}{a} = \rho$, so that ρ is the radius vector expressed in parts of the major semi-axis of the orbit. Further, let A, D denote the apparent Right Ascension and Declination of the planet's centre; α, δ those of the satellite; Δ the linear distance of the observer from the planet. If the origin of the coordinates is placed in the satellite instead of in the centre of the planet, the strict formulæ become somewhat simplified, but for the purpose α and δ must be known. In case approximately correct values of $\alpha - A$ and $\delta - D$ are available, like those of the three outer satellites of *Saturn*, published for some years past in the *Monthly Notices*, or if they can be derived directly from the observations, it will be worth while to take advantage of the favourable circumstance.

As most of the observed positions of satellites consist of measures of position-angles p and distances s, or are given in polar-coordinates, the formulæ for their proper computation may first be taken into consideration.

If the Earth's position, as seen from the satellite, is referred to the plane of the satellite's orbit by the longitude $U + 180°$ and latitude B, and if P is the position-angle of the pole of the orbit (in Right Ascension $N - 90°$ and Declination $90° - J$) at the geocentric place of the satellite, the values of U, B, P are found from those of N, J, α, δ by means of the equations:

$$\cos B \sin U = \cos\delta \sin(\alpha - N) \cos J + \sin\delta \sin J$$
$$\cos B \cos U = \cos\delta \cos(\alpha - N)$$
$$\sin B = \cos\delta \sin(\alpha - N) \sin J - \sin\delta \cos J$$
$$\cos B \sin P = -\cos(\alpha - N) \sin J$$
$$\cos B \cos P = \sin\delta \sin(\alpha - N) \sin J + \cos\delta \cos J$$

or by equivalent formulæ.

The satellite's orbital longitude $u = Q + v$, and also ρ being deduced from the mean anomaly μ, either by the usual formulæ:

$$\epsilon - e\omega^{\circ} \sin\epsilon = \mu \qquad \omega^{\circ} = 57^{\circ}.296 = \frac{1}{\text{arc } 1^{\circ}}$$
$$\rho \sin v = \sin\epsilon \cos\phi$$
$$\rho \cos v = \cos\epsilon - e$$

or by the series for the equation of the centre $v-\mu$ and $\log \rho$, the values of the position-angle p and of the distance s corresponding to u are found by means of the equations

$$\sin\sigma\sin(p-\mathrm{P}) = \sin(u-\mathrm{U})$$
$$\sin\sigma\cos(p-\mathrm{P}) = \cos(u-\mathrm{U})\sin\mathrm{B}$$
$$\sin s = \frac{a}{\Delta}\cdot\rho\sin\sigma$$

in which σ is the arc on the sphere between the geocentric and planetocentric place of the satellite, or, without the introduction of σ, by

$$\sin s\sin(p-\mathrm{P}) = \frac{a}{\Delta}\rho\sin(u-\mathrm{U})$$
$$\sin s\cos(p-\mathrm{P}) = \frac{a}{\Delta}\rho\cos(u-\mathrm{U})\sin\mathrm{B}$$

The position-angle p thus computed refers to the Declination-circle passing through the satellite. In case the observed position-angle refers to the Declination-circle passing through the point midway between the planet's centre and the satellite, a correction $= -\frac{1}{2}s\sin p\tan\delta$ must, when a strict comparison is required, be added to the computed p.

If A, D are substituted for α, δ in the formulæ for U, B, P, so that their ensuing values refer to the centre of the planet instead of the satellite, the correction of the computed p will be $= +\frac{1}{2}s\sin p\tan\mathrm{D}$, the computed σ (which must be taken between 90° and 180°, when $\cos(u-\mathrm{U})$ is negative) represents the arc between the geocentric place of the planet and the planetocentric place of the satellite, and the value of s must be found indirectly from the equation

$$\sin s = \frac{a}{\Delta}\rho\sin(\sigma-s),$$

if the employment of the equations

$$\Delta_1\sin s\sin(p-\mathrm{P}) = a\rho\sin(u-\mathrm{U})$$
$$\Delta_1\sin s\cos(p-\mathrm{P}) = a\rho\cos(u-\mathrm{U})\sin\mathrm{B}$$
$$\Delta_1\cos s = a\rho\cos(u-\mathrm{U})\cos\mathrm{B}+\Delta,$$

in which Δ_1 denotes the distance of the satellite from the earth, is to be avoided.

The equations of condition for correcting the assumed elements of the orbit gain in clearness and simplicity by the introduction of

$$\sin\sigma\sin\tau = \sin\mathrm{B}$$
$$\sin\sigma\cos\tau = \cos\mathrm{B}\sin(u-\mathrm{U})$$
$$\cos\sigma = \cos\mathrm{B}\cos(u-\mathrm{U})$$

which serve, at the same time, as a partial control upon the equations for finding p:

$$\sin\sigma\sin(p-\mathrm{P}) = \sin(u-\mathrm{U})$$
$$\sin\sigma\cos(p-\mathrm{P}) = \cos(u-\mathrm{U})\sin\mathrm{B}$$
$$\sin s = \frac{a}{\Delta}\,.\,\rho\sin\sigma$$

The differentiation of these equations gives:

$$\sin\sigma dp = \sin\tau\,.\,d(u-\mathrm{U}) - \cos\tau\cos(u-\mathrm{U})\,d\mathrm{B} + \sin\sigma d\mathrm{P}$$
$$d\sigma = \cos\tau\,.\,d(u-\mathrm{U}) + \sin\tau\cos(u-\mathrm{U})\,d\mathrm{B}$$

But

$$\cos\mathrm{B}d\mathrm{U} = -\cos\mathrm{U}\sin\mathrm{B}d\mathrm{J} - \cos\mathrm{P}\cos\delta d\mathrm{N}$$
$$\cos\mathrm{B}d\mathrm{P} = -\cos\mathrm{U}d\mathrm{J} + \sin\mathrm{U}\sin\mathrm{J}d\mathrm{N}$$
$$d\mathrm{B} = +\sin\mathrm{U}d\mathrm{J} - \cos\mathrm{U}\sin\mathrm{J}d\mathrm{N}.$$

Hence, after some reductions:

$$\sin\sigma dp = \sin\tau du - \cos\tau\sin u dJ + (\cos\sigma\cos p\cos\delta - \sin\sigma\sin\delta)dN$$
$$d\sigma = \cos\tau du + \sin\tau\sin u d\mathrm{J} + \sin p\cos\delta d\mathrm{N}$$
$$\rho\sin\sigma\,.\,\omega^{\circ}\frac{ds}{s} = \rho\cos\sigma d\sigma + \sin\sigma\omega^{\circ}d\rho + \rho\sin\sigma\omega^{\circ}\,.\,\frac{da}{a}.$$

The differential expressions of u, the satellite's true longitude, with respect to its mean longitude $u_{\circ}$ and Q and ϕ, are:

$$\rho du = \frac{\cos\phi}{\rho}du_{\circ} + \left(\rho - \frac{\cos\phi}{\rho}\right)d\mathrm{Q} + (1+\rho\sec^2\phi)\sin v\,.\,\cos\phi d\phi$$
$$= (1+e\cos v)\sec\phi du_{\circ} - \{2\cos v\}\tan\phi d\mathrm{Q} + \{2\sin v\}\cos\phi d\phi$$

in which the coefficients in { } brackets denote the following equivalents, among which the most convenient, according to facilities at hand, may be selected:

$$\{2\cos v\} = \cos v + \cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi$$
$$= \frac{(2+e\cos v)\cos v + e + \tan\frac{1}{2}\phi\cos^2\phi}{1+e\cos v}$$
$$= \left(2 - \frac{e\cos v}{1+e\cos v}\right)\cos v + \frac{e\cos\phi + \tan\frac{1}{2}\phi}{1+e\cos v}$$
$$= (1+\rho\sec^2\phi)\cos v + \rho(\tan\phi + \tan\tfrac{1}{2}\phi\sec^2\phi)$$
$$\{2\sin v\} = \sin v + \sin\epsilon\sec\phi$$
$$= \frac{2+e\cos v}{1+e\cos v}\sin v$$
$$= \left(2 - \frac{e\cos v}{1+e\cos v}\right)\sin v$$
$$= (1+\rho\sec^2\phi)\sin v.$$

The substitution of these expressions and of

$$\omega^{\circ}d\rho = \tan\phi\sin v du_{\circ} - \sin v\,.\,\tan\phi d\mathrm{Q} - \cos v\,.\,\cos\phi d\phi$$

in the equations for $\sin\sigma dp$ and ds leads to the equations be-

tween the variations of the computed coordinates and the variations of the assumed elements

$$\rho \sin \sigma \delta p = \sin \tau \,.\, \frac{\cos \phi}{\rho} \,.\, \delta u_\circ$$
$$- \sin \tau \,.\, \{2 \cos v\}\ \omega^\circ (\tan \phi + \delta e) \sin \delta Q$$
$$+ \sin \tau \,.\, \{2 \sin v\} \omega^\circ [(\tan \phi + \delta e) \cos \delta Q - \tan \phi]$$
$$- \rho \cos \tau \sin u \delta J$$
$$+ \rho (\cos \sigma \cos p \cos \delta - \sin \sigma \sin \delta)\, \delta N$$

$$\rho \sin \sigma \,.\, \omega^\circ \frac{\delta s}{s} = (\cos \sigma \cos \tau \,.\, \frac{\cos \phi}{\rho} + \tan \phi \sin \sigma \sin v) \delta u_\circ$$
$$- (\cos \sigma \cos \tau \{2 \cos v\} + \sin \sigma \sin v)\, \omega^\circ (\tan \phi + \delta e) \sin \delta Q$$
$$+ (\cos \sigma \cos \tau \{2 \sin v\} - \sin \sigma \cos v) \omega^\circ [(\tan \phi + \delta e) \cos \delta Q - \tan \phi]$$
$$+ \rho \cos \sigma \sin \tau \sin u \,.\, \delta J$$
$$+ \rho \cos \sigma \sin p \cos \delta \,.\, \delta N$$
$$+ \rho \sin \sigma \,.\, \omega^\circ \frac{\delta a}{a}.$$

In order to guard against some possible oversight or error in the interpretation of the variations connected with the eccentricity if they are written in the form $\tan \phi\, \delta Q$ and $\cos \phi\, \delta \phi$, I have substituted in these equations for

$$\tan \phi \delta Q \ \ldots\ldots\ \omega^\circ (\tan \phi + \delta e) \sin \delta Q$$

and for

$$\cos \phi \delta \phi \ldots\ldots\ \omega^\circ [(\tan \phi + \delta e) \cos \delta Q - \tan \phi]$$

expressions which represent the true meaning of these variations, and will lead to right deductions of the corrections of Q and e in all cases. If not thus interpreted the expressions $\tan \phi\, \delta Q$ and $\cos \phi\, \delta \phi$ or δe may obviously be misleading, and may give very incorrect results whenever the assumed values of Q and e require more than small corrections.

If, instead of δQ and δe, the variations of suitable functions of Q and e are to be introduced, the equations for $\rho\, du$ and $d\rho$ suggest the advantage of selecting for the purpose not $e \sin Q$ and $e \cos Q$, but E sin Q and E cos Q, where E is such a function of e or ϕ that

$$\tan \phi d\mathrm{E} = \mathrm{E} de = \mathrm{E} \cos \phi d\phi.$$

Putting accordingly

$$\mathrm{E} = e \,.\, \sec^2 \tfrac{1}{2}\phi \,.\, c^{-2 \sin^2 \frac{1}{2}\phi} = 2 \tan \tfrac{1}{2}\phi \,.\, c^{-2 \sin^2 \frac{1}{2}\phi} \qquad (c \text{ basis of nat. log.})$$

the differential coefficients of u and ρ with respect to E sin Q and E cos Q will be

$$\rho du = \ldots - \frac{(2 + e \cos v) \cos u + (e \cos \phi + \tan \frac{1}{2}\phi) \cos Q}{1 + e \cos v} \,.\, \omega^\circ \,.\, \frac{\tan \phi}{\mathrm{E}} \,.\, d(\mathrm{E} \sin Q)$$
$$+ \frac{(2 + e \cos v) \sin u + (e \cos \phi + \tan \frac{1}{2}\phi) \sin Q}{1 + e \cos v} \,.\, \omega^\circ \,.\, \frac{\tan \phi}{\mathrm{E}} \,.\, d(\mathrm{E} \cos Q),$$

$$d\rho = \ldots - \sin u \,.\, \frac{\tan \phi}{\mathrm{E}} \,.\, d(\mathrm{E} \sin Q) - \cos u \,.\, \frac{\tan \phi}{\mathrm{E}} \,.\, d(\mathrm{E} \cos Q).$$

Hence the eccentricity terms in the equations of condition for δp and δs become

$$\rho \sin\sigma\delta p = \ldots - \sin\tau \cdot \frac{(2+e\cos v)\cos u + (e\cos\phi + \tan\tfrac{1}{2}\phi)\cos Q}{1+e\cos v} \cdot \omega^{\circ} \cdot \frac{\tan\phi}{E} \cdot \delta(E\sin Q)$$

$$+ \sin\tau \cdot \frac{(2+e\cos v)\sin u + (e\cos\phi + \tan\tfrac{1}{2}\phi)\sin Q}{1+e\cos v} \cdot \omega^{\circ} \cdot \frac{\tan\phi}{E} \cdot \delta(E\cos Q)$$

$$\rho \sin\sigma \frac{\delta s}{s} = \ldots - (\cos\sigma\cos\tau \cdot \frac{(2+e\cos v)\cos u + (e\cos\phi + \tan\tfrac{1}{2}\phi)\cos Q}{1+e\cos v} + \sin\sigma\sin u) \cdot \frac{\tan\phi}{E} \cdot \delta(E\sin Q)$$

$$+ (\cos\sigma\cos\tau \cdot \frac{(2+e\cos v)\sin u + (e\cos\phi + \tan\tfrac{1}{2}\phi)\sin Q}{1+e\cos v} - \sin\sigma\cos u) \cdot \frac{\tan\phi}{E} \cdot \delta(E\cos Q).$$

Though in this way the corrections of the assumed values of Q and e might be properly found and mistaken deductions avoided, the way would be needlessly circuitous; for, by substituting in the foregoing expressions for $\delta(E\sin Q)$ and $\delta(E\cos Q)$ their values

$$(E+\delta E)\sin(Q+\delta Q) - E\sin Q, \text{ and}$$
$$(E+\delta E)\cos(Q+\delta Q) - E\cos Q,$$

and by eliminating Q, the expressions are obtained which are already given in the equations of condition, and which, while of equal strictness, are obviously preferable on account of their greater simplicity and directness. Moreover, they allow the motion of the apses, if approximately known, to be easily taken into account.

The equations of condition have yet to be so adjusted that the differences between the observed and the computed co-ordinates are measured by the same unit. It will depend on the character of the measured position-angles and distances whether it is preferable to express the differences in seconds of arc or in units corresponding to an arc of one degree on a circle of suitable radius, and also how far the ensuing rules are better modified or altered.

If $\sin a_{\circ} = \frac{a}{\Delta_{\circ}}$, where $\Delta_{\circ}$ is some conveniently fixed value of Δ (not differing greatly from the average of the distances Δ at which observations are taken), and if $\frac{\Delta_{\circ}}{\Delta} = \nu$, the angular distances s of the satellite will be

$$s = a_{\circ}\nu \cdot \rho\sin\sigma.$$

Making

$$\frac{a_0}{57{\cdot}3} = \kappa$$

the adopted unit, the equations of conditions will be

$$\begin{aligned}
\frac{s}{a_0}\delta p = &+\nu \sin\tau \,.\, \frac{\cos\phi}{\rho} \,.\, \delta u_0 \\
&-\nu \sin\tau \,.\, \{2\cos v\} \,.\, \omega^\circ\, (\tan\phi + \delta e) \sin \delta Q \\
&+\nu \sin\tau \,.\, \{2\sin v\} \,.\, \omega^\circ\, [(\tan\phi + \delta e)\cos\delta Q - \tan\phi \\
&-\nu\rho\cos\tau \,.\, \sin u \,.\, \delta J \\
&+\nu\rho\,(\cos\sigma\cos p\cos\delta - \sin\sigma\sin\delta) \,.\, \delta N \\
\frac{\delta s}{\kappa} = &+\nu(\cos\sigma\cos\tau \,.\, \frac{\cos\phi}{\rho} + \tan\phi\sin\sigma\sin v) \,.\, \delta u_0 \\
&-\nu(\cos\sigma\cos\tau \,.\, \{2\cos v\} + \sin\sigma\sin v) \,.\, \omega^\circ\, (\tan\phi + \delta e)\sin\delta Q \\
&+\nu(\cos\sigma\cos\tau \,.\, \{2\sin v\} - \sin\sigma\cos v) \,.\, \omega^\circ\, [(\tan\phi + \delta e)\cos\delta Q - \tan\phi] \\
&+\nu\rho\cos\sigma\sin\tau \,.\, \sin u \,.\, \delta J \\
&+\nu\rho\cos\sigma\sin p\cos\delta \,.\, \delta N \\
&+\frac{s}{a_0} \,.\, \frac{\delta a_0}{\kappa}.
\end{aligned}$$

If the differences between the observed and computed coordinates are expressed in seconds of arc, the equations become

$$\begin{aligned}
\frac{s}{57{\cdot}3}\delta p = &+\nu\sin\tau \,.\, \frac{\cos\phi}{\rho} \,.\, \kappa\delta u_0 \\
&-\nu\sin\tau \,.\, \{2\cos v\} \,.\, a_0\,(\tan\phi + \delta e)\sin\delta Q \\
&+\nu\sin\tau \,.\, \{2\sin v\} \,.\, a_0\,[(\tan\phi + \delta e)\cos\delta Q - \tan\phi] \\
&-\nu\rho\cos\tau \,.\, \sin u \,.\, \kappa\delta J \\
&+\nu\rho\,(\cos\sigma\cos p\cos\delta - \sin\sigma\sin\delta) \,.\, \kappa\delta N \\
\delta s = &+\nu\,(\cos\sigma\cos\tau \,.\, \frac{\cos\phi}{\rho} + \tan\phi\sin\sigma\sin v) \,.\, \kappa\delta u_0 \\
&-\nu\,(\cos\sigma\cos\tau \,.\, \{2\cos v\} + \sin\sigma\sin v) \,.\, a_0\,(\tan\phi + \delta e)\sin\delta Q \\
&+\nu\,(\cos\sigma\cos\tau \,.\, \{2\sin v\} - \sin\sigma\cos v) \,.\, a_0\,[(\tan\phi + \delta e)\cos\delta Q - \tan\phi] \\
&+\nu\rho\cos\sigma\sin\tau \,.\, \sin u \,.\, \kappa\delta J \\
&+\nu\rho\cos\sigma\sin p\cos\delta \,.\, \kappa\delta N \\
&+\frac{s}{a_0} \,.\, \delta a_0.
\end{aligned}$$

The formulæ are here written out in full. Abbreviated notations for the quantities which are to be determined and for their coefficients will, of course, be substituted in actual computations.

The preceding equations are valid for orbits of any assumed

ellipticity. For assumed circular elements they get considerably simplified, and become

$$\begin{aligned}
\frac{s}{a_0}\delta p &= +\nu \sin\tau \,.\, \delta u \\
&\quad -\nu \sin\tau \,.\, \cos u \,.\, 2\phi \sin \mathrm{Q} \\
&\quad +\nu \sin\tau \,.\, \sin u \,.\, 2\phi \cos \mathrm{Q} \\
&\quad -\nu \cos\tau \,.\, \sin u \,.\, \delta \mathrm{J} \\
&\quad +\nu (\cos\sigma \cos p \cos\delta - \sin\sigma \sin\delta) \,.\, \delta \mathrm{N} \\
\frac{\delta s}{\kappa} &= +\nu \cos\sigma \cos\tau \,.\, \delta u \\
&\quad -\nu (\cos\sigma \cos\tau \,.\, \cos u + \tfrac{1}{2} \sin\sigma \sin u) \,.\, 2\phi \sin \mathrm{Q} \\
&\quad +\nu (\cos\sigma \cos\tau \,.\, \sin u - \tfrac{1}{2} \sin\sigma \cos u) \,.\, 2\phi \cos \mathrm{Q} \\
&\quad +\nu \cos\sigma \sin\tau \,.\, \sin u \,.\, \delta \mathrm{J} \\
&\quad +\nu \cos\sigma \sin p \cos\delta \,.\, \delta \mathrm{N} \\
&\quad + \frac{s}{a_0} \,.\, \frac{\delta a_0}{\kappa}.
\end{aligned}$$

They must be multiplied by κ if the differences are to be expressed in seconds.

The orbital longitudes u, U, Q employed in the preceding formulæ are reckoned from the node N, and this reckoning is most suitable whenever the inclination J is considerable. But if J is only of moderate amount, as in the case of the orbits of *Saturn's* satellites, the determination of the nodal point is considerably uncertain, and this uncertainty rapidly increases with the decrease of J. By adding the longitude of N to the longitudes reckoned from N the uncertainty is evaded, and the longitudes in the orbit start from a properly fixed point.

Putting $u+\mathrm{N}=l$ and $\mathrm{U}+\mathrm{N}=\mathrm{L}$, the formulæ for finding p and the sin and cos of σ and τ require merely the substitution of $l-\mathrm{L}$ for $u-\mathrm{U}$:

$$\begin{aligned}
&\sin\sigma \sin(p-\mathrm{P}) = \sin(l-\mathrm{L}) && \sin\sigma \sin\tau = \sin \mathrm{B} \\
&\sin\sigma \cos(p-\mathrm{P}) = \cos(l-\mathrm{L}) \sin \mathrm{B} && \sin\sigma \cos\tau = \cos \mathrm{B} \sin(l-\mathrm{L}) \\
&\sin s = \frac{a}{\Delta} \,.\, \rho \sin\sigma \text{ or } s = a_0\nu \,.\, \rho \sin\sigma && \cos\sigma = \cos \mathrm{B} \cos(l-\mathrm{L}).
\end{aligned}$$

The differential expressions for $\sin\sigma dp$ and $d\sigma$ become by the substitution of $dl-d\mathrm{N}$ for du

$$\begin{aligned}
\sin\sigma \, dp &= \sin\tau \,.\, dl - \cos\tau \sin u \,.\, d\mathrm{J} + (\cos\sigma \cos p \cos\delta - \sin\sigma \sin\delta - \sin\tau) d\mathrm{N} \\
d\sigma &= \cos\tau \,.\, dl + \sin\tau \sin u \,.\, d\mathrm{J} + (\sin p \cos\delta - \cos\tau) d\mathrm{N}.
\end{aligned}$$

But

$$\begin{aligned}
\cos\sigma \cos p \cos\delta - \sin\sigma \sin\delta &= \sin\tau \cos \mathrm{J} + \cos\tau \cos u \sin \mathrm{J} \\
\sin p \cos\delta &= \cos\tau \cos \mathrm{J} - \sin\tau \cos u \cos \mathrm{J}.
\end{aligned}$$

Hence

$$\begin{aligned}
\sin\sigma dp &= \sin\tau \,.\, dl + [\cos\tau \cos(l-\mathrm{N}) - \tan\tfrac{1}{2}\mathrm{J} \sin\tau] \sin \mathrm{J} \,.\, d\mathrm{N} - \cos\tau \sin(l-\mathrm{N}) d\mathrm{J} \\
d\sigma &= \cos\tau \,.\, dl - [\sin\tau \cos(l-\mathrm{N}) + \tan\tfrac{1}{2}\mathrm{J} \cos\tau] \sin \mathrm{J} \,.\, d\mathrm{N} + \sin\tau \sin(l-\mathrm{N}) d\mathrm{J}
\end{aligned}$$

The equations of condition between the variations of the coordinates and the variations of functions of the elements become accordingly

$$\frac{s}{57{\cdot}3}\delta p = +\nu\sin\tau\,.\,\frac{\cos\phi}{\rho}\,.\,\kappa\delta l_0$$
$$-\nu\sin\tau\,.\,\{2\cos v\}\,.\,a_0(\tan\phi+\delta e)\sin\delta(\mathrm{Q}+\mathrm{N})$$
$$+\nu\sin\tau\,.\,\{2\sin v\}\,.\,a_0[(\tan\phi+\delta e)\cos\delta(\mathrm{Q}+\mathrm{N})-\tan\phi]$$
$$+\nu\rho[\cos\tau\cos(l-\mathrm{N})-\tan\tfrac{1}{2}\mathrm{J}\sin\tau]\,.\,a_0\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\sin\delta\mathrm{N}$$
$$-\nu\rho\cos\tau\sin(l-\mathrm{N})\,.\,a_0[\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\cos\delta\mathrm{N}-\sin\mathrm{J}]$$

$$\delta s = +\nu\Big(\cos\sigma\cos\tau\,.\,\frac{\cos\phi}{\rho}+\tan\phi\sin\sigma\sin v\Big)\,.\,\kappa\delta l_0$$
$$-\nu(\cos\sigma\cos\tau\,.\,\{2\cos v\}+\sin\sigma\sin v)\,.\,a_0(\tan\phi+\delta e)\sin\delta(\mathrm{Q}+\mathrm{N})$$
$$+\nu(\cos\sigma\cos\tau\,.\,\{2\sin v\}-\sin\sigma\cos v)\,.\,a_0[(\tan\phi+\delta e)\cos\delta(\mathrm{Q}+\mathrm{N})-\tan\phi]$$
$$-\nu\rho\cos\sigma[\sin\tau\cos(l-\mathrm{N})+\tan\tfrac{1}{2}\mathrm{J}\cos\tau]\,.\,a_0\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\sin\delta\mathrm{N}$$
$$+\nu\rho\cos\sigma\,.\,\sin\tau\sin(l-\mathrm{N})\,.\,a_0[\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\cos\delta\mathrm{N}-\sin\mathrm{J}]$$
$$+\frac{s}{a_0}\,.\,\delta a_0$$

In order to guard against incorrect deductions of the corrections of the node and inclination, which might occur, if the terms sin J . δN and δJ were kept without proper interpretation, I have substituted for

$$\kappa\,.\,\sin\mathrm{J}\,.\,\delta\mathrm{N}\;\ldots\;a_0\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\sin\delta\mathrm{N}$$

and for

$$\kappa\,.\,\delta\mathrm{J}\;\ldots\;a_0[\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\cos\delta\mathrm{N}-\sin\mathrm{J}]$$

expressions which, as in the analogous case of the eccentricity terms, represent the true meaning of these terms, and cannot mislead. By this simple substitution all the advantages for the correctness of the deductions are secured, which otherwise might be gained by the introduction of the variations of tan $\frac{1}{2}$J sin N and tan $\frac{1}{2}$J cos N, but without their attending drawbacks, as mathematical readers may easily satisfy themselves.

The quantities

$$\kappa\delta l_0,\;\; a_0(\tan\phi+\delta e)\sin\delta(\mathrm{Q}+\mathrm{N}),\;\; a_0[(\tan\phi+\delta e)\cos\delta(\mathrm{Q}+\mathrm{N})-\tan\phi],$$
$$a_0\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\sin\delta\mathrm{N},\;\; a_0[\Big(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\Big)\cos\delta\mathrm{N}-\sin\mathrm{J}]$$

which are to be treated as the unknown quantities, will in the formulæ for actual computation, of course, be represented by single letters.

For assumed circular elements the equations of condition become

$$\frac{s}{57{\cdot}3}\delta p = +\nu \sin\tau \,.\, \kappa\delta l$$
$$-\nu \sin\tau \,.\, \cos l \,.\, 2\kappa\phi \sin(\mathrm{Q}+\mathrm{N})$$
$$+\nu \sin\tau \,.\, \sin l \,.\, 2\kappa\phi \cos(\mathrm{Q}+\mathrm{N})$$
$$+\nu\,[\cos\tau\cos(l-\mathrm{N}) - \tan\tfrac{1}{2}\mathrm{J}\sin\tau]\,.\, a_0\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\sin\delta\mathrm{N}$$
$$-\nu\cos\tau\sin(l-\mathrm{N})\,.\,a_0\left[\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\cos\delta\mathrm{N} - \sin\mathrm{J}\right]$$

$$\delta s = +\nu\cos\sigma\cos\tau \,.\, \kappa\delta l_0$$
$$-\nu(\cos\sigma\cos\tau \,.\, \cos l + \tfrac{1}{2}\sin\sigma\sin l) \,.\, 2\kappa\phi\sin(\mathrm{Q}+\mathrm{N})$$
$$+\nu(\cos\sigma\cos\tau \,.\, \sin l - \tfrac{1}{2}\sin\sigma\cos l) \,.\, 2\kappa\phi\cos(\mathrm{Q}+\mathrm{N})$$
$$-\nu\cos\sigma\,[\sin\tau\cos(l-\mathrm{N}) + \tan\tfrac{1}{2}\mathrm{J}\cos\tau]\,.\,a_0\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\sin\delta\mathrm{N}$$
$$+\nu\cos\sigma \,.\, \sin\tau\sin(l-\mathrm{N}) \,.\, a_0\left[\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\cos\delta\mathrm{N} - \sin\mathrm{J}\right]$$
$$+\frac{s}{a_0}\,.\,\delta a_0.$$

In turning from the consideration of the proper treatment of measured polar coordinates to that of rectangular coordinates, it is obviously expedient to take first the special case of *Saturn's* satellites, when their rectangular coordinates parallel to the axes of the ring are either measured or estimated. In measuring the distances from the axes it is essential that the position-angles in which the measures are taken should be distinctly stated, so that if they differ from the computed position-angles of the axes which represent the fundamental plane, to which the orbits of the satellites are to be referred, the necessary corrections may be applied.

If L, B, P have the same significance as before in reference to the assumed plane of the ring, the coordinates x'', y'' parallel to the axes of the ring are

$$s\sin(p-\mathrm{P}) = x'' = a_0\nu \,.\, \rho\sin(l-\mathrm{L})$$
$$s\cos(p-\mathrm{P}) = y'' = a_0\nu \,.\, \rho\cos(l-\mathrm{L})\sin\mathrm{B}$$

$$\delta x'' = +\nu\,[\cos(l-\mathrm{L}) + e\cos\mathrm{M}] \,.\, \kappa\sec\phi\delta l_0$$
$$-\nu\,[\cos(l-\mathrm{L})(\cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi) + \cos\mathrm{M}] \,.\, a_0(\tan\phi+\delta e)\sin\delta(\mathrm{Q}+\mathrm{N})$$
$$+\nu\,[\cos(l-\mathrm{L})\sin\epsilon\sec\phi + \sin\mathrm{M}] \,.\, a_0[(\tan\phi+\delta e)\cos\delta(\mathrm{Q}+\mathrm{N}) - \tan\phi]$$
$$-\nu\rho \,.\, \tan\tfrac{1}{2}\mathrm{J}\cos(l-\mathrm{L}) \,.\, a_0\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\sin\delta\mathrm{N}$$
$$+\frac{x''}{a_0}\,.\,\delta a_0.$$

$$\delta y'' = -\nu\sin\mathrm{B}\,[\sin(l-\mathrm{L}) - e\sin\mathrm{M}] \,.\, \kappa\sec\phi\delta l_0$$
$$+\nu\sin\mathrm{B}\,[\sin(l-\mathrm{L})(\cos\epsilon\cos\phi + \tan\tfrac{1}{2}\phi) - \sin\mathrm{M}] \,.\, a_0(\tan\phi+\delta e)\sin\delta(\mathrm{Q}+\mathrm{N})$$
$$-\nu\sin\mathrm{B}\,[\sin(l-\mathrm{L})\sin\epsilon\sec\phi + \cos\mathrm{M}] \,.\, a_0[(\tan\phi+\delta e)\cos\delta(\mathrm{Q}+\mathrm{N}) - \tan\phi]$$
$$+\nu\rho\,[\cos\mathrm{B}\cos(l-\mathrm{N}) - \tan\tfrac{1}{2}\mathrm{J}\sin\mathrm{B}\sin(l-\mathrm{L})] \,.\, a_0\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\sin\delta\mathrm{N}$$
$$+\nu\rho\cos\mathrm{B}\sin(l-\mathrm{N}) \,.\, a_0\left[\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\cos\delta\mathrm{N} - \sin\mathrm{J}\right]$$
$$+\frac{y''}{a_0}\,.\,\delta a_0.$$

For convenience M is put instead of $L-(Q+N)$. If the eccentric anomaly ϵ is not available

$$\frac{\cos v + e\cos\phi + \tan\frac{1}{2}\phi}{1+e\cos v} \text{ must be put for } \cos\epsilon\cos\phi + \tan\frac{1}{2}\phi$$

and

$$\frac{\sin v}{1+e\cos v} \text{ for } \sin\epsilon\sec\phi.$$

For assumed circular elements the equations of condition become

$$\begin{aligned}
\delta x'' = {} & +\nu\cos(l-L)\,.\,\kappa\delta l \\
& -\nu[\cos(l-L)\cos l+\cos L]\,.\,a_0 e\sin(Q+N) \\
& +\nu[\cos(l-L)\sin l+\sin L]\,.\,a_0 e\cos(Q+N) \\
& -\nu\tan\tfrac{1}{2}J\cos(l-L)\,.\,a_0\left(\sin J+\frac{\delta J}{\omega^\circ}\right)\sin\delta N \\
& +\frac{x''}{a_0}\,.\,\delta a_0. \\
\delta y'' = {} & -\nu\sin B\,.\,\sin(l-L)\,.\,\kappa\delta l \\
& +\nu\sin B[\sin(l-L)\cos l-\sin L]\,.\,a_0 e\sin(Q+N) \\
& -\nu\sin B[\sin(l-L)\sin l+\cos L]\,.\,a_0 e\cos(Q+N) \\
& -\nu[\cos B\cos(l-N)-\tan\tfrac{1}{2}J\sin B\sin(l-L)]\,.\,a_0\left(\sin J+\frac{\delta J}{\omega^\circ}\right)\sin\delta N \\
& +\nu\cos B\sin(l-N)\,.\,a_0\left[\left(\sin J+\frac{\delta J}{\omega^\circ}\right)\cos\delta N-\sin J\right] \\
& +\frac{y''}{a_0}\,.\,\delta a_0.
\end{aligned}$$

In case the x ordinates are not measured micrometrically, but estimated by comparison with certain definite points of the major axis of the ring, the proper unit for x and for the semi-axis of the satellite's orbit will obviously be the semi-diameter of *Saturn's* equator. If (a) is the length of the semi-axis expressed in such units, the computed

$$x=(a)\rho\sin(l-L)$$

and the equations of condition require merely the substitution of (a) for $a_\circ\,\nu$.

Though the range of such estimations of x is limited, there is the great advantage that they can be made without micrometer, and even with a rather unsteady instrument, and that when taken with due care and circumspection at the right times, they yield in accuracy only to the best micrometrical measurements. Observations which fix the times of the conjunctions of the satellites with the limbs of the ball and the ends of the ring in the four quadrants are of great value, not only in investigations

of the orbital longitudes of the satellites, but also in examining the dimensions of the ball and ring.

If the observed positions of the satellite in reference to the planet's centre consist of differences of Right Ascension and Declination, or are given in equivalent rectangular coordinates, the frequently used method for their computation may be followed, which Bessel employs in his investigation of the orbit of *Titan* ("Astr.-Nachr.," Nos. 193–195). His auxiliary angles f, F, g, G, h, H, referred to the geocentric place of the planet A, D, may be found by means of the formulæ *

$$\sin f \sin F = -\sin(A-N) \qquad \sin g \sin(G-F) = -\sin(D+E)$$
$$\sin f \cos F = +\cos(A-N)\cos J \qquad \sin g \cos(G-F) = +\cos(D+E)\cos f$$
$$\cos f = -\cos(A-N)\sin J \qquad \cos g = +\cos(D+E)\sin f$$
$$\sin f \sin E = -\sin(A-N)\sin J \qquad \sin h \sin(H-F) = +\cos(D+E)$$
$$\sin f \cos E = +\cos J \qquad \sin h \cos(H-F) = -\sin(D+E)\cos f$$
$$\cos h = +\sin(D+E)\sin f.$$

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* The values of b, B, c, C in the expressions for a planet's or comet's heliocentric coordinates referred to the equator—

$$x = r \sin a \sin(A' + v),$$
$$y = r \sin b \sin(B' + v),$$
$$z = r \sin c \sin(C' + v),$$

may be found, in a similar manner, by means of the formulæ

$$\sin a \sin E = +\sin i \cos \Omega \qquad \sin b \sin(B-A) = -\cos(E+\epsilon)$$
$$\sin a \cos E = +\cos i \qquad \sin b \cos(B-A) = +\sin(E+\epsilon)\cos a$$
$$\cos a = +\sin i \sin \Omega \qquad \cos b = -\sin(E+\epsilon)\sin a$$
$$\sin a \sin A = +\cos \Omega \qquad \sin c \sin(C-A) = -\sin(E+\epsilon)$$
$$\sin a \cos A = -\sin \Omega \cos i \qquad \sin c \cos(C-A) = -\cos(E+\epsilon)\cos a$$
$$A' = A + \omega \qquad \cos c = +\cos(E+\epsilon)\sin a$$

so that the $B' + v$ and $C' + v$ in the expressions for y and z are obtained by simply adding B − A and C − A to the values of $A' + v$.

Are these convenient formulæ for b, B, c, C not to be found anywhere in print? Considering that they refer to computations of such frequent occurrence, it is not easy to assume that they have not been pointed out before.

In case the inclination i is of moderate amount, it may be worth while, for greater accuracy, to determine A by computing

$$\sin \nu = \tan \tfrac{1}{2} i \,.\, \sin \Omega \,.\, \sin E$$

which makes $$A = \Omega + 90^\circ - \nu.$$

If formulæ for control are required,

$$\sin b \sin B = \sin \Omega \cos \epsilon$$
$$\sin c \sin C = \sin \Omega \sin \epsilon$$
$$\tan(C-B) = \frac{\cos a}{\cos b \cos c}$$

may serve the purpose.

The positions of the satellite are then found by

$$\Delta_{,}\sin s\sin p = r\sin f\sin(\mathrm{F}+u) \qquad = \Delta\,.\,\xi$$
$$\Delta_{,}\sin s\cos p = r\sin g\sin(\mathrm{G}+u) \qquad = \Delta\,.\,y$$
$$\Delta_{,}\cos s = \Delta + r\sin h\sin(\mathrm{H}+a) = \Delta\,.\,(1+\zeta);$$

or, if the difference between s and $\omega''\tan s$ is neglected, the rectangular coordinates by

$$x'' = \frac{\omega''}{\Delta}\,.\,\frac{r}{1+\zeta}\sin f\sin(\mathrm{F}+u)$$
$$y'' = \frac{\omega''}{\Delta}\,.\,\frac{r}{1+\zeta}\sin g\sin(\mathrm{G}+u).$$
$$\omega'' = \frac{1}{\text{arc }1'}$$

By the substitution of α, δ for A, D in the computations of the auxiliary angles f, F, g, G, or by computing their values, not for the apparent place of the planet, but for that of the satellite, the divisor $1+\zeta$ will be got rid of, and the formulæ become simplified. If that course is adopted they will be

$$x'' = \omega''\sin s\sin p = \omega''\cos\mathrm{D}\sin(\alpha-\mathrm{A}) \qquad = a_0\nu\rho\,.\,\sin f\sin(\mathrm{F}+u)$$
$$y'' = \omega''\sin s\cos p = \omega''\sin(\delta-\mathrm{D}) - x''.\tan\tfrac{1}{2}(\alpha-\mathrm{A})\sin\delta = a_0\nu\rho\,.\,\sin g\sin(\mathrm{G}+u)$$

or, if $\mathrm{F}' = \mathrm{F}+\mathrm{Q}$, $\mathrm{G}' = \mathrm{G}+\mathrm{Q}$,

$$x'' = a_0\nu\sin f\,.\,\rho\sin(\mathrm{F}'+v)$$
$$y'' = a_0\nu\sin g\,.\,\rho\sin(\mathrm{G}'+v).$$

The equations of condition are

$$\begin{aligned}
\delta x'' = {} & +\nu\sin f\,[\cos(\mathrm{F}'+v)+e\cos\mathrm{F}']\,.\,\kappa\sec\phi\,\delta l_0 \\
& -\nu\sin f\,[\cos(\mathrm{F}'+v)\,(\rho\cos v\cos\phi + e\cos\phi + \tan\tfrac{1}{2}\phi) + \cos\mathrm{F}']\,. \\
& \qquad a_0(\tan\phi+\delta e)\sin\delta\,(\mathrm{Q}+\mathrm{N}) \\
& +\nu\sin f\,[\cos(\mathrm{F}'+v)\,.\,\rho\sin v\sec^2\phi - \sin\mathrm{F}']\,.\,a_0[(\tan\phi+\delta e)\cos\delta\,(\mathrm{Q}+\mathrm{N}) \\
& \qquad -\tan\phi] \\
& +\nu\rho\,.\,\tan\tfrac{1}{2}\mathrm{J}\,.\,\cos(l-\mathrm{N}+\alpha-\mathrm{N})\,.\,a_0\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\sin\delta\mathrm{N} \\
& +\nu\rho\sin(l-\mathrm{N})\cos f\,.\,a_0[\sin\left(\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\cos\delta\mathrm{N} - \sin\mathrm{J}] \\
& +\frac{x''}{a_0}\,\delta a_0
\end{aligned}$$

$$\begin{aligned}
\delta y'' = {} & +\nu\sin g\,[\cos(\mathrm{G}'+v)+e\cos\mathrm{G}']\,.\,\kappa\sec\phi\,.\,\delta l_0 \\
& -\nu\sin g\,[\cos(\mathrm{G}'+v)\,(\rho\cos v\cos\phi + e\cos\phi + \tan\tfrac{1}{2}\phi) + \cos\mathrm{G}']\,. \\
& \qquad a_0(\tan\phi+\delta e)\sin\delta\,(\mathrm{Q}+\mathrm{N}) \\
& +\nu\sin g\,[\cos(\mathrm{G}'+v)\,.\,\rho\sin v\sec^2\phi - \sin\mathrm{G}']\,.\,a_0[(\tan\phi+\delta e)\cos\delta(\mathrm{Q}+\mathrm{N}) \\
& \qquad -\tan\phi] \\
& -\nu\rho\,[\cos(l-\mathrm{N})\cos\delta + \tan\tfrac{1}{2}\mathrm{J}\sin(l-\mathrm{N}+\alpha-\mathrm{N})\sin\delta]\,. \\
& \qquad a_0\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\sin\delta\,\mathrm{N} \\
& +\nu\rho\sin(l-\mathrm{N})\,.\,\cos g\,.\,a_0[\left(\sin\mathrm{J}+\frac{\delta\mathrm{J}}{\omega^\circ}\right)\cos\delta\mathrm{N} - \sin\mathrm{J}] \\
& +\frac{y''}{a_0}\,.\,\delta a_0.
\end{aligned}$$

If the assumed elements are circular, and if $\mathrm{F}_{,} = \mathrm{F}-\mathrm{N}$, $\mathrm{G}_{,} = \mathrm{G}-\mathrm{N}$, the coordinates become

$$x'' = a_0\nu\sin f\,.\,\sin(\mathrm{F}_{,}+l)$$
$$y'' = a_0\nu\sin g\,.\,\sin(\mathrm{G}_{,}+l)$$

and the equations of condition

$$\delta x'' = +\nu \sin f \cos(F_{,}+l) \,.\, \kappa\delta l_0$$
$$-\nu \sin f\,[\cos(F_{,}+l)\cos l+\cos F_{,}] \,.\, a_0 e \sin(Q+N)$$
$$+\nu \sin f\,[\cos(F_{,}+l)\sin l-\sin F_{,}] \,.\, a_0 e \cos(Q+N)$$
$$+\nu \tan \tfrac{1}{2}J \,.\, \cos(l-N+\alpha-N) \,.\, a_0\left(\sin J+\frac{\delta J}{\omega^{\circ}}\right)\sin \delta N$$
$$+\nu \sin(l-N)\cos f \,.\, a_0\left[\sin\left(J_0+\frac{\delta J}{\omega^{\circ}}\right)\cos \delta N-\sin J\right]$$
$$+\frac{x''}{a_0}\,.\,\delta a_0$$

$$\delta y'' = +\nu \sin g \cos(G_{,}+l) \,.\, \kappa\delta l_0$$
$$-\nu \sin g\,[\cos(G_{,}+l)\cos l+\cos G_{,}] \,.\, a_0 e \sin(Q+N)$$
$$+\nu \sin g\,[\cos(G_{,}+l)\sin l-\sin G_{,}] \,.\, a_0 e \cos(Q+N)$$
$$-\nu\,[\cos(l-N)\cos\delta+\tan\tfrac{1}{2}J\sin(l-N+\alpha-N)\sin\delta] \,.\, a_0 \sin\left(J+\frac{\delta J}{\omega^{\circ}}\right)\sin \delta N$$
$$+\nu \rho \sin(l-N)\cos g \,.\, a_0 \sin\left[\left(J+\frac{\delta J}{\omega^{\circ}}\right)\cos \delta N-\sin J\right]$$
$$+\frac{y''}{a_0}\,.\,\delta a_0$$

The auxiliary angles are found by the formulæ

$$\sin f \sin E = +\sin J \sin(\alpha-N) \qquad \sin g \sin(G-F) = -\sin(E+\delta)$$
$$\sin f \cos E = +\cos J \qquad \sin g \cos(G-F) = -\cos(E+\delta)\cos f$$
$$\cos f = -\sin J \cos(\alpha-N) \qquad \cos g = +\cos(E+\delta)\sin f$$
$$\sin f \sin F = -\sin(\alpha-N)$$
$$\sin f \cos F = +\cos(\alpha-N)\cos J$$

or F−N by

$$\sin(F-N+\alpha) = \tan\tfrac{1}{2}J \,.\, \cos(\alpha-N) \,.\, \sin E.$$

The values of B and P used in the formulæ for polar-coordinates are

$$\cos B \sin P = -\cos(\alpha-N)\sin J = \cos f$$
$$\cos B \cos P = +\cos(E+\delta)\sin f = \cos g$$
$$\sin B = -\sin(E+\delta)\sin f$$

On the Orbit of OΣ 400. By J. E. Gore.

Some measures of this close binary star, made in 1885 with the 23-inch Refractor of the Princeton (U.S.A.) Observatory, and kindly sent me by Professor Young, show that the companion has described about 190° of its apparent ellipse since its discovery by O. Struve in 1844.

I have computed the orbit, and find the following provisional elements :—

Elements of OΣ 400.

P = 170·37 years	Ω = 146° 20′
T = 1882·09	λ = 43° 30′
e = 0·669	a = 0″·59
γ = 36° 58′	μ = −2°·113

The following table shows a comparison between these elements, and the observations used in the calculation of the orbit:—

Epoch.	Observer.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
1844·83	O. Struve	336°·9	335°·23	+ 1°·67	0″·65	0″·74	− 0″·09
1845·69	,,	334·7	334·57	+ 0·13	0·67	0·73	− 0·06
1846·68	,,	333·1	333·73	− 0·63	0·60	0·72	− 0·12
1851·81	,,	328·8	329·32	− 0·52	0·56	0·67	− 0·11
1853·23	,,	324·6	327·70	− 3·10	0·59	0·65	− 0·06
1853·89	Dawes	320·55	327·00	− 6·45	0·65	0·64	+ 0·01
1854·69	O. Struve	320·5	326·17	− 5·67	0·62	0·63	− 0·01
1858 59	,,	320·6	321·53	− 0·93	0·62	0·57	+ 0·05
1860·10	,,	319·3	319·40	− 0·10	0·62	0·55	+ 0·07
1861·62	,,	318·1	317·13	+ 0·97	0·62	0·53	+ 0·09
1865·51	Dembowski	311·2	310·0	+ 1·20	"difficillissima"	0·45	—
1865·94	,,	310·3	309·06	+ 1·24	—	0·44	—
1868·55	,,	301·6	302·50	− 0·90	"difficile" "cert. oblunga"	0·39	—
1870·51	,,	302·0	296·27	+ 5·73	"difficillissima"	0·35	—
1871·57	,,	300·4	292·1	+ 8·30	"ovale"	0·32	—
1872·57	,,	"Semplice?"			—	0·30	—
1873·91	,,	289·3	280·2	+ 9·10	"oblunga?" "molta incerta"	0·27	—
1874·50	,,	287·8	276·3	+ 11·5	"oblunga?"	0·26	—
1875·67	Schiaparelli	267·9	267·70	+ 0·20	0·33	0·23	+ 0·10
1877·59	Dembowski	"Semplice; aria ottima"			—	0·20	—
1878 647	Burnham	"Single with all powers" of $18\frac{1}{2}$-in. refractor				0·17	—
1885·715	Young	149·5	143·0	+ 6·5	0·3 "est"	0·23	—
1885·731	,,	139·5	143·0	− 3·5	> 0·25	0·23	> + 0·02

Considering that the errors in the measures of so close a pair are necessarily considerable, the above comparison may be considered as fairly satisfactory.

The position of the binary is for 1880—

R.A. $20^h\ 6^m\ 14^s$

$+\ 43°\ 35'$

Magnitudes, 7·3, 7·7, according to Burnham.

A Working Catalogue of "Red" Stars. By G. F. Chambers.

The following catalogue is the outcome of many hundreds of observations, extending over (more particularly) a period of about seventeen years (1870–86), though not a few of the stars were examined in an unmethodical fashion during several years previous to 1870. Between 1870 and 1881 the telescope employed was a 4-inch Refractor by Cooke, but all the observations since 1884 have been made with a 6-inch Refractor by Grubb, almost always charged with an eyepiece of very low power, having a field of $1\frac{1}{4}°$. The observations of colour cited with the name "Brodie" appended were made by my cousin, Mr. C. G. Brodie, of Fernhill, I.W., with an $8\frac{1}{2}$-inch Refractor, in the years 1884–86. He has seen most of the stars which I have observed, but I have quoted from his notes only, as a rule, in cases where we did not agree in our estimates of colour.

The existing catalogues of red stars from which contributions have been levied are the following: but I have not limited myself to these, having been in the habit for many years past of making notes of all red stars wheresoever mentioned :—

1804. Lalande, J. de. *Tables des Étoiles Rouges.*
(*Connaissances des Temps*, An. xv. p. 378.)

1822. De Zach, Baron. *Étoiles Rouges.*
(*Corresp. Ast.* vol. vii. p. 298.)

1847. Herschel, Sir J. *Ruby-coloured, or very Intense Red Stars.*
(*Cape Observations*, p. 448.)

1866. Schjellerup, H. C *Catalog der rothen, isolirten Sterne.*
(*Ast. Nach.* vol. lxvii. No. 1591, June 18, 1866; Addenda, vol. lxviii. No. 1613, October 30, 1866.)

1872. Schmidt, J. F. J. *Verzeichniss rothgelber Sterne.*
(*Ast. Nach.* vol. lxxx. No. 1902, September 5, 1872.)

1874. Schjellerup, H. C. *Zweiter Catalog der rothen, isolirten Sterne.*
(*Vierteljahrsschrift der Astronomischen Gesellschaft*, vol. ix.)

1876. Burnham, S. W. *Catalogue of Red Double Stars.*
(*Month. Not.* vol. xxxvi. p. 331, 1876.)

1877. Birmingham, J. *Observations and Catalogue of Red Stars.*
(*Trans. Roy. Irish Acad.* vol. xxvi. p. 249, 1877.)

1877. Fearnley. *Des Étoiles colorées.*
(*Ast. Nach.* vol. lxxxix. No. 2121, March 27, 1877.)

1879. LINDEMANN, E. *Verzeichniss von 42 neuen rothen Sternen.*
(*Bulletin de l'Acad. de St.-Pétersbourg*, vol. xxv. p. 155.)
1882. LINDEMANN, E. *Zweites Verzeichniss neuer rother Sternen.*
(*Bulletin de l'Acad. de St.-Pétersbourg*, vol. xxviii. p. 278.)
1885. ESPIN, Rev. T. E. *Some New Red Stars.*
(*Journal of Liverpool Ast. Soc.* vol. iii. p. 82, March 1885.)
1886. ESPIN, Rev. T. E. *Some New Red and Orange-red Stars.*
(*Month. Not. R.A.S.* vol. xlvi. p. 293, March 1886.)

This catalogue makes no pretence to being exhaustive; it must not be regarded as more than it professes to be, namely, a working list of the best of the red stars, almost always excluding known variables, many of which are notoriously red in colour. My reason for excluding known variables was this: their inclusion would have been a trap to observers using this list for the purpose for which alone it is designed, namely, to facilitate the study of any red stars which are always within reach of their instruments (regard being had to the season of the year and the latitude of the place of observation).

And in another sense this catalogue is not exhaustive; it only includes stars of decided colour and not less than $8\frac{1}{2}$ magnitude. It may be that my eyes are not so sensitive to red hues as many other eyes are, but I certainly have often considered that many of my predecessors in the observation of red stars have greatly exaggerated the colours they have ascribed to particular objects. Taking the so-called "red" stars all round, my opinion is that a more generally accurate generic term for them would be "orange" stars, and that very few indeed rise to real "red," and that less than a dozen can be truly termed "carmine" or "ruby." These remarks seem requisite by way of caution in order to guard inexperienced observers from being disappointed when they come to examine for themselves stars described by Sir W. Herschel, Sir J. Herschel, Schmidt, and others, as "red" or "very red." There can be no doubt that these observers saw many stars to possess a redder tinge than they have since been found to exhibit. This may have been in the case of the Herschels some effect of their metallic mirrors, or of their eyesight, or may have been due to both causes combined; and, speaking generally, it may be said that the vast majority of the stars here catalogued are "orange" more than anything else. I would for this purpose define as "orange" the colour exhibited by the gilding inside articles of silver plate newly gilt.

A few, and only a few, explanations are requisite as to the principles on which the catalogue has been constructed. In column 1 one asterisk indicates objects of particular interest; two asterisks objects of very special and remarkable interest.

Column 2 gives the progressive numbers, if any, from Birmingham's catalogue. The places in columns 4 and 5 have been taken from the best authorities available, brought up to 1890. I have freely consulted the catalogues of Stone and Yarnall, and the Armagh catalogue, and the two Radcliffe catalogues, besides Birmingham's. The places of such of the stars as are to be found in the *Nautical Almanac* for 1890, or the *American Ephemeris* for 1889, are taken from those works respectively. Espin's places are as given by him, but brought up from 1885 to 1890. As, however, he only quotes the *Durchmusterung* to the nearest minute of Right Ascension and Declination, my 1890 places of his stars will be less exact than in the case of all the other stars. The magnitudes in column 6 are in all cases from Pickering's *Harvard Photometry*, where the star was to be found in that important and interesting record; one of the most valuable contributions to practical astronomy that has appeared for many years. The magnitudes of many of the remaining stars have been determined by Mr. C. G. Brodie by Dawes's method. In the column headed "Colour; Remarks" the information given within inverted commas has been generally taken from published sources, although the authority is not in all cases mentioned. All other details not in inverted commas reproduce the results of my own original work. The stars examined by myself amount to 589, out of a total of 719, being virtually nearly all those visible in England.

It would not be difficult to offer many interesting and suggestive reflections as the result of a prolonged study of the red stars; but one would soon be apt to drift away into speculations, which, however attractive to some minds, would not only be mere speculations, but would relate to matters quite beyond the domain of astronomy proper. I will therefore here only record the well-known facts that many of the stars certainly variable are red, and that many of the red stars have, since they were first noted as such, been found to be variable. The instances that could be cited in support of these statements are very numerous, and it is quite impossible for the coincidence between the redness of the colour and the variability of the light of several hundred stars to be accidental. What it means I do not presume to suggest. Amongst observers who have paid much attention to the colours of stars a foremost place must be given to the late Mr. J. Birmingham, of Tuam, whose labours in the matter of red stars added much to our knowledge of stellar colours. It was he who remarked that "a space of the heavens, including the Milky Way, between Aquila, Lyra, and Cygnus, seems so peculiarly favoured by red and orange stars that it might not inaptly be called 'the red region,' or 'the red region of Cygnus'; and, although the chances of finding a star of any stated colour must, of course, be greater among the countless multitudes of the Milky Way than elsewhere, still its other portions visible in this latitude show no such special richness in red stars."

One other remark of Birmingham's will be useful in this

place: "The red stars seem as liable to change of tint as to change of magnitude; and, although modifications of colour may have been remarked without any striking change of size, still I have observed that as a red variable increases it grows paler, and that it reddens deeper towards the minimum. Schmidt has made the same observation, and it well accords with the fact that all the very red stars are telescopic, and none of them visible to the naked eye. This is noteworthy, and seems difficult of explanation; but the cause, as I would suggest, may be found in the quality of light received from the object. As we do not see the stars by their discs, but by the amount of their light, according to apparent magnitude, it seems evident that the very red stars, shining with only a part of the components of white light, must appear less bright than the white stars, and seem therefore generally small. According to this view, it might be assumed that the few naked eye reddish stars would appear larger if white; so that Aldebaran, Betelgeuse, &c., if of that colour, might be rivals of Sirius. The redness of a star has given rise to the singular conceit that it shows a cooling down, or, as we might say, an approach to a final snuffing out of the luminary; but one might think that the fact of periodic variation of tint in many of the red stars ought to go far in disproving this proposition."

The reason why I have not included stars below the 9th magnitude is that where one is dealing with a star which is near the *minimum visibile* of a telescope estimations of colour are apt to become very imaginary in many cases. I say this without any desire to impute bad faith to an observer who might talk about "magnitude 13, colour red"; but I nevertheless should view his chromatic notions with a certain amount of distrust. I do not believe that it is, as a rule, possible to assign colour to stars of the 12th or lesser magnitudes, except a very large telescope indeed happens to be employed in viewing them.

The question has often been discussed as to how far flat surface diagrams of colour are of any use as standards of comparison for coloured stars. For my part I am disposed to question their usefulness, notwithstanding that I caused Smyth's well-known diagram of coloured discs to be reproduced for the new edition of the *Cycle of Celestial Objects*. I have often thought of trying whether a series of transparent discs of coloured glass, arranged in gradations of colour, and mounted in a frame sufficiently portable to be held in the hand by the observer whilst his eye is looking through the telescope, might not be used for comparisons of star colours. Probably the chief difficulty would be the obtaining of a sufficiently pure white light wherewith to illuminate the discs. If this were got over by the use of electricity or otherwise, I think such an apparatus might be effective and trustworthy, whilst it need not be very expensive nor very cumbersome.

The notes bearing the names of Bellamy and Robinson were sent to me in MS. by Mr. E. J. Stone, Director of the Radcliffe Observatory, Oxford.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Co'our; Remarks.
1	1	— Cassiopeiæ	0 3 38	+63 20·4	9	Reddish.
2		Cassiopeiæ	0 9 5	+65 29·8	8½	Reddish.
3	3	ι Ceti	0 13 49	− 9 26·0	3½	Golden yellow.
*4	4	— Andromedæ	0 14 5	+44 5·9	8	Fine fiery red.
5		— Cassiopeiæ	0 29 9	+67 19·2	6¾	Pale red.
6		— Cassiopeiæ	0 31 21	+67 2·2	7¼	Reddish.
**7	8	δ Andromedæ	0 33 26	+30 15·6	3¼	Golden yellow.
8	9	α Cassiopeiæ	0 34 16	+55 56·0	2¼	Fine yellow.
9	10	β Ceti	0 38 4	−18 35·4	2	Golden yellow.
10		— Cassiopeiæ	0 44 25	+61 10·8	6¼	Fiery orange.
11		— Cassiopeiæ	0 46 18	+69 21·9	7½	Orange.
12	11	— Cassiopeiæ	0 50 59	+67 5·9	8¾	Reddish.
13	12	2 Ursæ Minoris	0 53 39	+85 39·1	5	Golden yellow.
14	15	239 Groom. Cassiopeiæ	1 0 36	+52 54·5	6¼	Pale orange.
**15	16	η Ceti	1 3 3	−10 45·8	3½	Fine golden yellow. "? Var."
16	17	β Andromedæ	1 3 34	+35 2·2	2¼	Red.
***17	18	— Piscium	1 10 4	+25 11·2	7	Fiery red.
18	20	— Andromedæ	1 11 26	+47 7·1	7	Slight tinge of red.
19	23	— Cassiopeiæ	1 20 3	+65 30·4	7	Pale red.
20	24	R Sculptoris	1 21 55	−33 7·3	6	"Orange red" (J. Herschel). "Brilliant scarlet" (Gould).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
21		36 Espin Persei	1 26 1	+57 49·7	6	Deep orange.
22	26	— Cassiopeiæ	1 26 11	+60 5·3	9	Decided red, especially in contrast with neighbouring stars. Cl. = 103 M.
23	28	α Eridani	1 33 36	−57 47·7	1	"Red."
**24	29	ν Piscium	1 35 42	+ 4 55·8	5	Orange.
25	30	— Andromedæ	1 36 55	+50 3·7	7½	Decided red.
***26	31	— Cassiopeiæ	1 47 44	+69 39·8	8	Fiery red.
27		145 Espin Andromedæ	1 52 6	+44 52·6	8	"Red" (Espin).
28		— Ceti	1 54 59	+ 9 9·2	6	Reddish.
29	33	— Persei	1 55 46	+54 41·7	8	Pale red.
30	34	γ Andromedæ	1 57 8	+41 48·1	2¼	Golden yellow.
***31	35	α Arietis	2 0 58	+22 56·5	2	Golden yellow.
32	36	— Ceti	2 1 8	+ 0 55·2	8	Slightly tinged.
33	37	60 Andromedæ	2 6 19	+43 42·9	5	Pale orange.
34	39	— Andromedæ	2 11 8	+44 41·9	8½	Reddish.
35	40	ο Ceti	2 13 47	− 3 28·7	Var.	Fiery red when approaching max.
36	41	— Persei	2 14 38	+56 37·9	9	Fiery red; about midway between the clusters 33 and 34 H VI.
37		65 Andromedæ	2 18 17	+49 46·7	5	Good orange.
38	45	Arg. +65 : 280 Cassiopeiæ	2 28 43	+65 16·5	6	Deep orange.
39		15 Trianguli	2 29 6	+34 12·5	Var. 5-8	Reddish orange.
40	46	— Andromedæ	2 30 26	+56 35·6	8	Pale red.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
41	47	855 Weisse Triang.	2 36 37	+31 57·5	7½	Red; neb. h 257 *p* (D'Arrest).
42	49	η Persei	2 42 40	+55 26·3	4	Orange.
43	50	5172 Lal. Persei	2 43 30	+57 52·6	7½	Red.
44	51	— Cassiopeiæ	2 47 23	+63 52·7	6½	Deep orange.
45	52	α Ceti	2 56 31	+ 3 39·5	2¾	Decided orange. "Orange red" (Robinson).
46	56	ω Persei	3 4 11	+39 11·6	4¾	Reddish yellow.
47	57	— Persei	3 4 54	+47 18·6	7	Pale red; 2 small red stars in the field *sf*.
48	59	1014 B. A. C. Horologii	3 9 46	−57 44·0	7	"Red."
49	58	6048 Lal. Eridani	3 10 11	− 9 10·6	7	Reddish; *p* ζ Erid. 18ˢ, and 4′ to the N.
50	60	W. B. III. 152 Eridani	3 10 55	− 6 7·9	7	"Reddish orange." "Ruddy" (Robinson).
51		*o* Tauri	3 18 53	+ 8 38·5	3½	Orange. "Red" (*Uran. Arg.*).
52	61	Arg. +54 : 685 Camelop.	3 21 49	+55 0·3	7½	Pale red.
53	62	σ Persei	3 22 49	+47 36·9	4½	Pale orange; contrasts well with white stars near.
54		Arg. +79 : 110 Camelop.	3 32 20	+79 58·4	7¾	"Red' (Espin).
*55	65	— Camelopardi	3 32 21	+62 17·4	7	Pale crimson.
56	66	— Tauri	3 36 2	+14 26·4	8	"Decided red."
57	67	— Persei	3 37 44	+53 33·6	8½	Red.
58	68	6921 Lal. Eridani	3 38 28	− 9 57·3	8	Reddish. ? Mag.
59	69	121 P. III. Camelop.	3 39 17	+65 10·9	6	Deep orange. Mag. 4·75 (*Uran. Oxon.*).
60	70	π Eridani	3 40 56	−12 26·8	4½	Reddish orange.
61		42 Espin Tauri	3 41 29	+24 38 9	7	Reddish orange.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
62	71	1204 B. A. C. Camelop.	3 47 43	+60 47·3	5¼	Pale orange.
63		γ Hydri.	3 48 57	−74 34·5	3½	"Deep yellow" (Williams).
64	72	7272 Lal. Eridani	3 49 53	−15 13·8	7	Orange; a larger star of the same colour *p*.
65		γ^1 Eridani	3 52 53	−13 49·3	3	Reddish orange. "Reddish" (Robinson).
66		220 P. III. Tauri	3 55 45	+ 9 41·4	6	Very pale orange. "Red" (*Uran. Arg.*)
67		Arg. +61 : 667 Camelop.	3 56 20	+61 29·6	7½	"Red" (Espin).
68		45 Espin Tauri	3 58 10	+12 12·0	7½	Reddish.
69		γ Reticuli	3 59 17	−62 28·0	5	"Deep yellow" (Williams).
70		Arg. +32 : 743 Persei	4 5 59	+32 14·8	7	Fiery orange.
71		47 Tauri	4 7 57	+ 8 59·1	5	Pale orange. "Red" (*Uran. Arg.*)
72		19 P. IV. Tauri	4 8 51	+ 9 43·8	5¼	Very pale orange. "Red" (*Uran. Arg.*)
73	73	8154 Lal. Eridani	4 15 14	− 6 30·4	6¼	Pale orange.
74	74	1342 B.A.C. Tauri	4 15 54	+20 33·4	6½	Fiery orange.
75		47 Espin Persei	4 17 4	+34 59·4	7½	Red.
76		45 Eridani	4 26 15	− 0 17·0	5	Deep orange. "Red" (*Uran. Arg.*)
77	79	8623 Lal. Eridani	4 28 10	−11 1·2	6½	Reddish.
*78		47 Eridani	4 28 53	− 8 27·5	5½	Reddish orange.
79	80	*e* (58) Persei	4 29 3	+41 2·2	4½	Golden.
80		W.B. IV. 585 Eridani	4 29 8	− 9 10·3	6	Fiery red. "Red" (*Uran. Arg.*)
**81	81	α Tauri	4 29 36	+16 17·2	1	Deep reddish orange.
82		146 Espin Aurigæ	4 37 27	+43 34·7	8	"Red" (Espin).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
83	82	— Aurigæ	4 38 7	+32 43·2	8¾	Pale crimson; an orange star *p*.
84		2 Espin Aurigæ	4 38 38	+32 14·7	8½	Red.
*85	83	1457 B.A.C. Camelop.	4 39 50	+67 58·4	7	Fiery red.
86		147 Espin Aurigæ	4 41 56	+34 48·4	8	"Red" (Espin).
*87	85	— Aurigæ	4 44 37	+28 20·2	8	Unmistakably crimson.
88		148 Espin Aurigæ	4 45 20	+38 19·0	8¼	"Very red" (Espin).
89	87	o^1 Orionis	4 46 19	+14 4·2	5½	Reddish orange.
90		149 Espin	4 47 12	+22 35·7	9	"Very red" (Espin).
*91	88	5 Orionis	4 47 38	+ 2 19·5	5¾	Deep orange. "Probably var."
92	89	236 P. IV. Orionis	4 49 51	+ 7 36·0	6	Deep golden yellow. "Orange red" (Brodie). Oxford R.A. = 4^h 48 9^m.
93	91	6 Aurigæ	4 52 49	+39 29·5	6½	Red.
94		3 Espin Aurigæ	4 53 3	+40 4·6	7½	Fiery red.
95		— Orionis	4 53 14	+12 40·0	8¾	Decided red. Position approximate.
96		4 Espin Aurigæ	4 53 44	+40 22·4	8	Reddish.
**97	94	R Leporis	4 54 36	−14 58·2	var.	Decided crimson.
98	93	ζ Aurigæ	4 54 47	+40 54·8	4	Fine deep orange. "Slightly orange" (*Uran. Oxon.*).
99		150 Espin Aurigæ	4 55 50	+38 54·8	8½	"Very red" (Espin).
100	95	276 P. IV. Orionis	4 56 11	+ 0 33·6	6¼	Pale orange. "? Var."
101		151 Espin Aurigæ	4 59 8	+34 42·3	8¼	"Red" (Espin).
102	96	899 H.P. Orionis	4 59 43	+ 1 1·6	7	Intense fiery red.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
103	97	ϵ Leporis	5 0 48	−22 31·1	3¼	Reddish orange. "? Var."
104	98	— Orionis	5 0 57	+ 0 24·2	9	Decided red.
105		152 Espin Aurigæ	5 4 15	+43 34·5	8	"Red" (Espin).
106	99	— Orionis	5 4 24	− 5 39·4	8	Full red.
107	100	9744 Lal. Orionis	5 4 26	− 0 42·1	6½	Golden yellow.
*108		— Leporis	5 6 38	−12 1·2	7½	Deep red.
109	102	Arg.−0 : 890 Orionis	5 9 1	− 0 41·2	7	Pale orange.
110		5 Espin Aurigæ	5 10 59	+40 20·6	7	Reddish orange.
*111		Arg.+40 : 1245 Aurigæ	5 11 12	+40 58·7	7½	Fiery red.
112		50 Espin Aurigæ	5 11 49	+35 40·3	8½	Fiery red.
113	103	— Aurigæ	5 12 30	+39 12·2	8	Pale red.
114		51 Espin Orionis	5 12 44	− 8 21·9	8½	Deep orange.
115		W.B. V. 266 Aurigæ	5 12 51	+41 0·3	5¼	Orange. "Slightly red" (*Uran. Oxon.*).
116	104	9919 Lal. Aurigæ	5 13 33	+34 9·2	8½	Red. "Reddish" (Brodie).
117		154 Espin Aurigæ	5 14 43	+32 23·4	9	"Very red" (Espin).
118	106	61 P.V. Orionis	5 16 27	+ 3 28·0	8	Red.
119	107	— Orionis	5 18 1	− 9 25·9	8½	Reddish. ? Var. in colour.
*120		53 Espin Orionis	5 19 50	−10 26·9	6	Reddish orange.
*121		52 Espin Aurigæ	5 20 7	+29 49·5	8	Almost pale ruby.
122		— Aurigæ	5 20 38	+35 13·3	7	Good orange. In cl. 39 H I VII.
123		6 Espin Aurigæ	5 22 39	+40 25·5	7½	Orange.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
124	109	S Orionis	5 23 34	− 4 47·0	var.	Reddish. In centre of small equilat. triangle of 3 stars.
125	110	31 Orionis	5 23 59	− 1 10·8	5-6	Deep orange. "Var."
126	111	119 Tauri	5 25 47	+18 30·7	4½	Pale red. "Good orange" (Brodie).
127		7 Espin Aurigæ	5 26 31	+41 2·3	7½	Reddish.
128		54 Espin Aurigæ	5 26 31	+32 40·0	7	Reddish orange; blue star 6½ mag. near.
129		10426 Lal. Orionis	5 26 59	− 0 3·8	8	Decided red.
130		156 Espin Orionis	5 27 16	+ 7 3·6	7¾	"Very red" (Espin).
131		— Tauri	5 28 3	+25 49·1	8½	Red.
132		10483 Lal. Orionis	5 28 29	− 1 32·4	7	Fiery red.
133	113	ϕ^2 Orionis	5 30 51	+ 9 14·1	4½	Pale orange.
134	114	— Orionis	5 30 58	+10 57·9	7	Reddish orange.
*135		124 Tauri	5 32 34	+23 15·5	7⅓	Quadruple star; A red.
136		β Doradûs	5 33 26	−62 33·7	3½	Reddish yellow (Williams).
137		157 Espin Aurigæ	5 33 37	+31 51·4	6½	"Orange red" (Espin).
*138		Arg. +31 : 1058 Aurigæ	5 34 59	+31 49·1	8½	Fiery red.
139	117	10743 Lal. Orionis	5 35 29	− 3 54·0	8	Reddish.
140	119	51 Orionis	5 36 48	+ 1 25·3	5¼	Golden yellow. "Slightly red" (*Uran. Oxon.*).
141	120	— Tauri	5 38 30	+24 22·3	8½	Decided red. ? Var.
142		55 Espin Aurigæ	5 38 41	+50 2·5	7	Reddish orange.
143		— Aurigæ	5 39 5	+30 39·4	7	Good red.
144	121	— Geminorum	5 39 6	+20 38·4	7½	Good red. "Deep orange red" (Brodie). "? Var."

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
145	124	— Pictoris	5 40 8	−46 30·5	8	"Vivid red" (J. Herschel).
146		56 Espin Aurigæ	5 40 33	+44 48·0	9	Red.
147		57 Espin Aurigæ	5 44 18	+32 5·8	6½	Fiery red. Near neb. 37 M.
148		11061 Lal. Orionis	5 44 22	+ 4 23·9	6	Deep orange. "Red" (*Uran. Arg.*).
149		Arg. + 32 : 1118 Aurigæ	5 46 6	+32 9·3	8½	Fiery red.
150		11 Espin Aurigæ	5 46 19	+40 21·8	8	Reddish.
151		58 Espin Aurigæ	5 46 42	+53 26·2	8	Red.
152		56 Orionis	5 46 44	+ 1 49·7	5	Reddish orange. "Red" (*Uran. Arg.*).
153	126	Arg. + 10 : 927 Orionis	5 48 7	+10 33·6	6½	Golden yellow.
154		Arg. + 20 : 1171 Orionis	5 49 5	+20 26·9	8	Reddish orange.
155	127	α Orionis	5 49 13	+ 7 23·1	1	Reddish orange.
156		Gore's *nova* Orionis	5 49 17	+20 9·6	6½	Fiery red. "Very red" (Robinson).
157	129	δ Aurigæ	5 50 28	+54 16·7	3¾	Lemon yellow.
158	130	π Aurigæ	5 51 45	+45 55·7	4½	Reddish orange. "Pale orange" (Brodie).
159	133	11451 Lal. Orionis	5 56 43	− 5 8·2	7	Reddish yellow. "Pale yellow" (Brodie).
160		13 Espin Aurigæ	6 0 5	− 5 51·0	7½	Reddish orange.
161		158 Espin Aurigæ	6 1 57	+47 43·1	8	"Red" (Espin).
*162	135	11684 Lal. Geminorum	6 4 3	+26 2·3	7½	Good red.
163		14 Espin Aurigæ	6 4 56	+32 43·3	6	Reddish.
164	136	Arg. + 21 : 1146 Geminorum	6 5 14	+21 53·6	7	Pale red.
165	137	Arg. + 22 : 1220 Geminorum	6 5 39	+22 55·8	7	Red.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
166		15 Espin Aurigæ	6 6 48	+33 16·3	7	Reddish orange.
167	139	η Geminorum	6 8 14	+22 32·3	3½	Reddish yellow.
168	141	Arg. + 39 : 1576 Aurigæ	6 9 3	+39 30·6	7½	Pale red. "? Var."
169	140	5 Monocerotis	6 9 30	− 6 14·4	4¼	Pale orange.
170		16 Espin Aurigæ	6 9 52	+39 53·7	7	Reddish.
171		17 Espin Aurigæ	6 10 2	+40 24·8	8½	Orange.
172		18 Espin Aurigæ	6 10 6	+39 30·6	7	Deep red.
173		62 Espin Aurigæ	6 10 14	+33 14·8	9	Very red.
174		2029 B.A.C. Geminorum	6 12 42	+23 19·1	7	Pale orange. "Magnificent spectrum."
175		1183 H.P. Canis Majoris	6 12 48	−16 46·3	5	Red.
176	142	12057 Lal. Orionis	6 13 47	+14 41·7	6	Reddish yellow.
177		12104 Lal. Orionis	6 14 29	− 2 53·2	5	Deep orange. "? Var." "Yellowish red" (Wickham).
178		160 Espin Aurigæ	6 15 24	+47 43·0	8½	"Red and probably var." (Espin).
179		12169 Lal. Canis Majoris	6 15 57	−11 45·8	7	Fiery red. "Red" (Robinson).
180	143	μ Geminorum	6 16 18	+22 34·1	3¼	Reddish orange.
181		161 Espin Geminorum	6 17 12	+25 4·2	9	"Very red" (Espin).
182		5 Lyncis	6 17 12	+58 28·7	5½	Orange.
183		3 Canis Majoris	6 18 5	−33 22·8	4	"Rich orange yellow" (Tupman).
*184	144	— Geminorum	6 19 11	+14 46·8	7	Full deep orange. "? Var." "Yellowish" (Brodie).
185	145	— Canis Majoris	6 19 16	−26 59·6	8	Pale crimson. "Intense ruby" (J. Herschel). "Orange red" (Brodie).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. °	Mag.	Colour; Remarks.
186		12359 Lal. Monocerotis	6 21 31	− 4 23·7	7	Deep orange.
187	147	— Monocerotis	6 24 56	− 2 56·8	8	Orange, or slightly reddish.
188		12524 Lal. Canis Majoris	6 25 25	−19 8·1	6½	Orange.
189		12545 Lal. Monocerotis	6 26 50	− 8 5·2	5½	Reddish orange. "Red" (*Uran. Arg.*).
190		20 Espin Monocerotis	6 28 19	− 2 59·8	7½	Orange. Position approximate.
**191	148	2139 B.A.C. Aurigæ	6 28 59	+38 32·0	6	Deep fiery red.
192		165 Espin Monocerotis	6 29 39	− 1 22·7	8¼	"Very red" (Espin).
193		21 Espin Aurigæ	6 31 2	+39 29·5	6½	Reddish orange.
194		ν^2 Canis Majoris	6 31 53	−19 9·7	4	Orange. "Red" (Robinson).
195	150	2196 B.A.C. Puppis	6 35 57	−52 50·0	6	"De color rojizo" (Moesta).
196		22 Espin Aurigæ	6 36 33	+40 44·2	7	Double: A 8, pale orange; B 10, blue; dist. 30″.
197	151	12907 Lal. Monocerotis	6 36 41	− 9 3·5	5½	Pale orange.
198	152	ϵ Geminorum	6 37 10	+25 14·3	3¼	Golden yellow. "? Var."
199		23 Espin Geminorum	6 38 36	+24 46 5	7½	Fiery orange.
200		17 Monocerotis	6 41 22	+ 8 9·4	5	Reddish yellow.
201	153	— Canis Majoris	6 42 18	−20 37·8	8	Pale red; near centre of cl. 41 M.
*202		13100 Lal. Monocerotis	6 42 22	− 8 52·4	5½	Reddish orange.
203		Arg. +61 : 915 Lyncis	6 43 10	+61 9 5	8	"Reddish" (Espin).
204	154	51 Cephei (Hev.)	6 48 46	+87 13·1	5¼	Full orange.
205	156	θ Canis Majoris	6 49 5	−11 54·1	4¼	Reddish orange. "Red" (Robinson).
206	157	o^1 Canis Majoris	6 49 34	−24 2·7	4	Deep orange.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
207	158	μ Canis Majoris	6 51 3	−13 54·0	5¼	Fiery red.
208		Arg. −8 : 1650 Monocerotis	6 52 49	− 8 52·6	7	"Orange red" (Espin).
209	161	2289 B.A.C. Puppis	6 53 20	−48 34·0	5½	"De color rojizo" (Mossta).
210	160	1245 Groom. Camelopardi	6 53 24	+70 53·5	6½	Pale red.
211		13627 Lal. Monocerotis	6 56 32	− 5 33·7	5½	Good orange. "? Var."
212		— Monocerotis	6 56 45	− 5 33·0	7	Pale orange.
213	162	22 Canis Majoris	6 57 20	−27 46·6	3½	Fiery red. "Reddish" (Brodie). "? Var."
214	163	— Monocerotis	6 57 40	− 8 11·2	8	Red. In cl. 50 M. "Reddish" (Brodie).
215		o^2 Canis Majoris	6 58 26	−23 40·4	3	Pale yellow.
**216	165	— Monocerotis	7 1 36	− 7 23·4	8	Crimson.
*217	166	— Canis Majoris	7 2 55	−11 45·3	7⅓	Decided red.
*218	167	R Canis Minoris	7 3 40	+10 11·9	7–10	Deep red. "Var."
219		2337 B.A.C. Geminorum	7 3 43	+13 44·3	6½	Deep orange.
220		δ Canis Majoris	7 3 55	−26 13·1	2	Reddish yellow. "Fine yellow" (Tupman).
221	168	2326 B.A.C. Camelopardi	7 7 56	+82 37·4	5¼	Reddish orange.
222	170	14038 Lal. Geminorum	7 8 59	+22 9·6	7	Orange, or pale red.
223		65 Espin Canis Majoris	7 12 0	−23 7·0	6½	Wide pair. A 6½, orange; B 7, blue. "A probably var." (Espin).
224		14184 Lal. Monocerotis	7 12 9	− 6 28·7	6½	Reddish orange.
225		L^2 Puppis	7 10 10	−44 27·7	5	"Red and var." (*Uran. Arg.*).
226	171	π Puppis	7 13 15	−36 54·0	3	"Beautiful orange" (Gore). "Very rich yellow" (Tupman).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
227	172	— Monocerotis	7 15 45	−10 10·9	8	Orange. In cl. h. 443
228		66 Aurigæ	7 16 31	+40 53·1	5½	Pale orange.
229	175	— Canis Majoris	7 18 28	−25 33·0	7	Red.
230		η Canis Majoris	7 19 44	−29 5·3	2½	Perhaps purplish. "? Purple" (Tupman). "Pale red" (Smyth).
231		171 Espin Lyncis	7 20 10	+46 11·4	8	"Red" (Espin).
232	176	14503 Lal. Canis Majoris	7 21 2	−20 44·1	8	Red.
233	178	14599 Lal. Monocerotis	7 24 4	−10 5·9	6	Reddish orange. "? Var."
234	179	σ Argûs	7 25 44	−43 4·9	5	"Red" (Schmidt).
*235		14776 Lal. Puppis	7 28 44	−14 17·0	5	Fiery red.
236	181	υ Geminorum	7 29 9	+27 8·7	4¼	Orange.
237		25 Espin Lyncis	7 29 38	+40 15·8	7	Reddish. The *p* star of a curious curve of 5 stars.
238		174 Espin Monocerotis	7 30 44	− 5 31·8	8	"Red" (Espin).
239		175 Espin Canis Minoris	7 30 44	+ 2 19·0	8¾	"Red with blue *comes*" (Espin).
240	182	1444 H. P. Geminorum	7 34 20	+23 17·3	6	Fiery red.
241		14952 Lal. Puppis	7 34 26	−16 35·5	6	Pale red.
242		26 Espin Canis Minoris	7 34 55	+ 4 19·4	7	Reddish orange. Position approximate.
243	183	27 Espin Geminorum	7 35 43	+29 5·2	8	Pale orange.
244		γ Monocerotis	7 36 0	− 9 17·7	4¼	Lemon yellow. "Reddish" (Robinson).
245		σ Geminorum	7 36 26	+29 9·2	4	Pale orange; a small red star *p* 39°.
246	186	15018 Lal. Monocerotis	7 37 4	−10 37·4	7½	Pale red.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
247	187	Arg. +5 : 1759 Canis Min.	7 37 33	+ 5 12·3	7	Pale red.
248	188	β Geminorum	7 38 35	+28 17·5	1	Pale yellow.
249	189	c Puppis	7 41 20	−37 42·1	4½	"Orange" (J. Herschel). The chief star in neb. h 3099.
250		28 Espin Lyncis	7 42 43	+40 2·9	7	Pale red.
251		176 Espin Canis Minoris	7 42 51	+ 5 42·0	9	"Very red" (Espin).
252		ξ Argûs	7 44 40	−24 35·0	3½	Good orange.
253	194	— Velorum	7 54 7	−49 41·5	8	"Brick red, inclining to orange" (J. Herschel).
254	195	— Carinæ	7 56 16	−60 31·2	8	"Orange;" in neb. h 3111; "near it a ruddy star" (J. Herschel).
255	196	2704 B.A.C. Lyncis	8 1 2	+58 34·7	6	Orange.
*256		β Cancri	8 10 33	+ 9 31·4	4	Reddish yellow.
257	201	2820 B.A.C. Puppis	8 19 13	−37 55·9	6	"De color rojizo" (Moesta).
258		ϵ Argûs	8 20 15	−59 9·3	2½	"Yellow; orange" (Tupman).
259		72 P. VIII. Puppis	8 20 18	−23 41·4	5½	Orange. "Orange" (Tupman).
260	205	17091 Lal. Hydræ	8 34 11	−19 21·1	6½	Red.
261		3449 Lac. Mali	8 35 5	−28 41·5	7	"Orange" (Gore). ? Place.
262	206	Arg. 8952 Mali	8 40 52	−27 47·9	8½	Full red.
263	208	— Puppis	8 46 14	−47 58·2	8½	"Ruby-coloured" (J. Herschel).
264	209	17497 Lal. Cancri	8 47 4	+19 44·3	8½	Reddish.
*265	211	17576 Lal. Cancri	8 49 11	+17 39·0	7	Pale crimson. "Rich orange red" (Brodie).
266	212	60 Cancri	8 49 56	+12 2·8	6–8	Reddish orange. "Probably var."

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
267	213	17624 Lal. Hydræ	8 50 1	−10 57·0	7½	Pale red: a blue star with a companion follows.
268		σ^1 Ursæ Majoris	8 58 48	+67 19·1	5¼	Deep orange.
269		ω Hydræ	9 0 11	+ 5 31·8	5½	Deep orange. "Red" (*Uran. Arg.*).
270	217	3121 B.A.C. Mali (κ Pyxis)	9 3 13	−25 24·4	4¾	Deep red.
271		λ Argûs	9 3 57	−42 59·3	2½	"Orange" (Tupman). "Blood red" (Pope).
272	218	Arg. +31 : 1946 Cancri	9 4 3	+31 24·9	6½	Fiery red. "Pale orange" (Brodie). "? Var."
273	219	π^2 Cancri	9 9 10	+15 24·0	5¾	Pale orange.
274		*g* Carinæ	9 13 5	−57 4·9	4½	"Deep orange red" (Tupman).
275	220	α (40) Lyncis	9 14 21	+34 51·4	3½	Reddish orange; a small blue star, *nf*, 12″.
276	221	Arg. +0 : 2499 Hydræ	9 14 58	+ 0 38·2	7	Reddish.
277	223	α Hydræ	9 22 11	− 8 10·9	2	Reddish yellow. "Possibly var."
278		18688 Lal. Hydræ	9 24 8	+20 16·0	6½	Fiery red. "Red" (Bellamy).
279		— Ursæ Majoris	9 25 11	+67 46·5	7	Orange. ? var. in colour. Decl. approximate.
280	224	λ Leonis	9 25 27	+23 27·3	4½	Reddish orange. "Slightly red" (*Uran. Oxon.*).
281		N Velorum	9 27 53	−56 32·9	3¾	"Very rich yellow" (Tupman).
282	225	— Carinæ	9 29 28	−62 18·5	8	"Sanguine red" (J. Herschel).
283	226	ι Hydræ	9 34 14	− 0 38·7	4¼	Orange.
284		ϵ Leonis	9 39 36	+24 16·8	3	Pale orange.
285		*l* Carinæ	9 41 13	−62 0·0	4½	"Yellow; orange" (Tupman).
286	228	R Leonis	9 41 39	+11 56·5	5–10	Pale crimson. "Var."
287	229	Ö-A (2) 10163 Hydræ	9 45 58	−22 30·2	7	Good red. "Distinctly red" (Bellamy).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
288		66 Espin Ursæ Majoris	9 49 0	+54 47·0	7	Dark orange. =2412 Rad.
289	230	— Velorum	9 50 57	−41 4·0	7½	"Scarlet" (J. Herschel).
290		29 Espin Leonis	9 51 46	+ 8 53·3	7	Reddish orange.
291	231	π Leonis	9 54 24	+ 8 34·3	5	Reddish orange. "Orange red" (Robinson).
292		19580 Lal. Sextantis	9 55 25	− 2 39·5	7	Reddish.
293	232	— Carinæ	9 56 25	−59 41·9	8½	"Scarlet" (J. Herschel).
294		19620 Lal. Hydræ	9 56 38	−23 16·5	7	Reddish.
295		19637 Lal. Sextantis	9 57 25	− 5 5·0	7	Reddish.
296	233	A (31) Leonis	10 2 4	+10 32·3	4½	Orange.
297	234	18 Sextantis	10 5 29	− 7 52·4	5¾	Reddish orange. "Orange" (Robinson). "? var."
298	235	— Antliæ	10 7 5	−34 46·7	7	"Scarlet" (J. Herschel).
299	236	— Carinæ	10 10 39	−60 8·4	9	"Ruby red" (J. Herschel).
300		30 Espin Ursæ Majoris	10 10 50	+42 0·9	6½	Fiery orange. "? var."
301		q Carinæ	10 13 24	−60 47·0	4	"Orange red" (Tupman).
302	237	γ Leonis	10 13 54	+20 23·8	2	Pale orange.
303		V Velorum	10 15 29	−54 28·6	5	"Red" (Tupman).
304	238	μ Ursæ Majoris	10 15 47	+42 4·2	3	Orange.
305	239	μ Hydræ	10 20 46	−16 16·5	4	Reddish orange.
306		s Carinæ	10 23 50	−58 10·6	4¼	"Yellow; orange" (Tupman).
307		4367 Lac. Carinæ	10 28 28	−72 39·3	5⅓	"Deep orange" (Tupman).
308	240	3630 B.A.C. Antliæ	10 30 20	−38 59·9	6½	"Orange; almost scarlet" (J. Herschel).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
309	241	r Carinæ	10 31 22	−56 59·3	5½	"De color rojizo" (Moesta). "Orange red" (Tupman).
**310	242	3637 B.A.C Hydræ	10 32 7	−12 48·8	5½	Fiery red. "Var. beyond question" (Gould).
311		t^2 Carinæ	10 34 34	−58 36·6	5	"Deep orange red" (Tupman).
312		— Ursæ Majoris	10 37 26	−67 9·3	6	Good pale red.
313		4435 Lac. Carinæ	10 38 25	−58 38·4	6	"Red" (Brisbane).
314	245	4446 Lac. Carinæ	10 39 21	−59 59·4	6	"Red" (Brisbane). "Deep orange red" (Tupman).
315	247	η Argûs	10 40 47	−59 6·5	6	"Orange" (Tupman). "Var."
316		μ Argûs	10 42 2	−48 50·3	3	"Very red" (Tupman).
317		20891 Lal. Sextantis	10 45 30	− 2 30·4	6½	Pale orange.
318	248	20918 Lal. Hydræ	10 46 16	−20 40·1	7	Pale crimson. "Very red" (Robinson).
319		u Carinæ	10 49 1	−58 16·1	4½	"Bright orange red" (Tupman).
320		— Leonis	10 50	+22 58	7	Orange. "Reddish yellow" (F. Brodie). Pos. approx.
321		31 Espin Leonis	10 52 45	+20 13·4	7	Reddish. ? Decl. 1° too little.
322	249	1917 H.P. Crateris	10 54 5	−15 45·8	6¼	Reddish.
323		α Crateris	10 54 27	−17 42·9	4½	Orange. "Red" (Robinson).
324	250	R Crateris	10 55 8	−17 37·6	8–9	Orange. "Var."
325	251	60 Leonis	10 56 27	+20 46·3	4½	Pale yellow.
326	252	α Ursæ Majoris	10 56 56	+62 20·7	2	Pale orange. "Var. in colour and perhaps in mag."
327	254	ψ Ursæ Majoris	11 3 29	+45 5·8	3	Reddish.
328		x Carinæ	11 3 53	−58 22·7	4¾	"Deep orange red" (Tupman).
329	255	— Chamæleontis	11 5 30	−81 11·5	8	"Ruby" (J. Herschel).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
330	256	72 Leonis	11 9 21	+23 41·7	5	Reddish yellow.
331	258	75 Leonis	11 11 37	+ 2 37·0	5½	Reddish orange.
**332	259	ν Ursæ Majoris	11 12 32	+33 41·7	3¾	Deep golden yellow.
333		δ Crateris	11 13 50	−14 11·0	3½	Reddish yellow. "Orange red" (Robinson).
334	261	87 Leonis	11 24 42	− 2 23·7	5	Orange.
335	262	λ Draconis	11 24 52	+69 56·3	4	Orange.
336		υ Leonis	11 31 19	− 0 13·0	4½	Reddish yellow.
337		ω Virginis	11 32 47	+ 8 44·5	5½	Orange. "Red" (*Uran. Arg.*).
338		22104 Lal. Crateris	11 34 16	−16 1·5	6	Red.
339	264	— Muscæ	11 34 34	−71 57·5	8½	"Fine ruby" (J. Herschel).
340	265	22122 Lal. Leonis	11 35 29	+25 51·5	8	Reddish yellow. Decl. only approximate.
341		32 Espin Ursæ Majoris	11 38 37	+49 7·6	8	Red.
342	266	χ Ursæ Majoris	11 40 24	+48 23·4	4	Golden.
343		4899 Lac. Muscæ	11 42 57	−66 12·9	5	"Red" (Tupman).
344	268	— Centauri	11 44 51	−56 34·0	8	"Orange;" about 10′ N. of neb. h 3365 (J. Herschel).
345	269	203 P. XI. Virginis	11 52 30	+ 4 6·5	8	Red.
346	270	1845 Groom. Ursæ Minoris	11 54 35	+81 28·1	6	Reddish.
347		5032 Lac. Crucis	12 2 40	−60 14·1	6	"Red" (Stone).
*348	272	ε Corvi	12 4 28	−22 0·5	3	Reddish orange. "Reddish" (Robinson). "? Var."
349		ε Muscæ	12 11 38	−67 20·9	5	"Orange red" (Tupman).
350		ε Crucis	12 15 26	−59 47·6	4	"Orange" (Tupman).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
351	276	17 Virginis	12 16 57	+ 5 55·2	7	Double, dist. 20″; A tinged; B red.
**352	277	— Virginis	12 19 38	+ 1 23·6	8¼	Good crimson. "? Var."
353		71 Ursæ Majoris	12 19 48	+57 23·3	5½	Good orange.
*354	279	γ Comæ Berenicis	12 21 27	+28 52·9	4¾	Deep golden yellow.
355	280	— Comæ Berenicis	12 23 40	+28 54·2	9	Certainly red. "Reddish tinge" (Brodie). "Purple" (J. Herschel).
356	281	— Virginis	12 24 44	+5 1·3	8½	Deep red.
357	282	γ Crucis	12 25 3	−56 29·6	2	"Clear orange yellow" (Gould).
358		4 Draconis	12 25 19	−69 48·4	5¼	Reddish orange.
359		23649 Lal. Virginis	12 33 49	− 5 29·7	6½	Reddish.
360	287	23736 Lal. Virginis	12 38 3	− 0 50·1	8½	Reddish.
*361	290	4287 B.A.C. Can. Venat.	12 39 57	+46 2·4	5½	Red. "Good orange" (Brodie). "Probably var."
362	291	— Crucis	12 40 58	−59 5·6	8½	"Most intense blood red" (J. Herschel).
363	295	κ Crucis	12 47 7	−59 45·2	7	In neb. h 3435; "central and largest star red."
364		ψ Virginis	12 48 38	− 8 56·4	5	"Red" (*Uran. Arg.*).
365	297	δ Virginis	12 50 3	+ 3 59·7	3¾	Lemon yellow. "Reddish yellow" (Robinson).
366	298	— Draconis	12 52 5	+66 35·3	7	Red.
367	299	24148 Lal. Comæ Berenicis	12 52 40	+18 21·6	8	Reddish. "Red tinge" (Brodie).
368		1944 Groom. Draconis	12 52 45	+67 50·2	6½	Orange.
369	300	36 Comæ Berenicis	12 53 30	+18 0·1	5	Reddish yellow. "Perhaps var."
370		9 Draconis	12 55 57	+67 11·5	5½	Good orange. ? Var.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
371		5460 Lac. Centauri	13 10 8	+44 7·4	7	"Red" (Stone).
372	302	σ Virginis	13 12 3	+ 0 3·0	5	Reddish yellow.
373		61 Virginis	13 12 40	−17 41·8	4¾	Pale orange.
374	303	γ Hydræ	13 12 54	−22 35·4	3½	Pale orange. "Red" (Brodie). "Perhaps var."
375		24769 Lal. Virginis	13 16 14	−13 50·5	7	Reddish tinge.
376	304	4479 B.A.C. Canum Venat.	13 18 54	+37 36·3	6½	Golden.
377	306	*i* Virginis	13 20 54	−12 8·1	5½	Reddish. "Yellowish red" (Brodie).
378	309	*l* (74) Virginis	13 26 14	− 5 41·1	5	Orange. "Red" (*Uran. Arg.*).
379	311	W.B. XIII. 596 Comæ Ber.	13 31 48	+25 10·2	6	Orange tinge.
380		25213 Lal. Virginis	13 34 2	−15 53·3	6½	Reddish.
381	312	83 Ursæ Majoris	13 36 34	+55 14·3	4¾	Golden. "? Var."
382	313	7561 Stone Hydræ	13 42 49	−27 49·0	6½	Red. "Splendid red."
383		25462 Lal. Virginis	13 44 11	−20 19·4	7	Reddish.
384	314	*v* Böotis	13 44 11	+16 20·6	4	Orange red. "Orange" (Brodie). "Probably var."
385	315	*i* (10) Draconis	13 48 11	+65 15·9	4¾	Golden. "Slightly red" (*Uran. Oxon.*).
386	316	3105 Rad. Canum Venat.	13 48 29	+40 52·8	7	Orange.
387	318	π Hydræ	14 0 6	−26 9·0	3½	Deep orange.
388		θ Centauri	14 0 13	−35 49·6	2	"Pale orange?" (Tupman).
389		95 Virginis	14 0 53	− 8 47·3	5¾	"Red" (*Uran. Arg.*).
390	319	— Centauri	14 1 19	−59 12·2	8	"Double: both stars brick red" (J. Herschel).
391	320	13 Böotis	14 4 10	+49 58·6	5½	Reddish.

Number, G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
*392	321	4700 B.A.C. Virginis	14 4 51	−15 47·0	5¼	Red.
393	322	κ Virginis	14 7 1	− 9 45·7	4¼	Reddish. "Orange red" (Robinson).
394	323	— Centauri	14 9 0	−59 24·0	7½	"Ruby or high orange" (J. Herschel).
395	324	4 Ursæ Minoris	14 9 17	+78 3·9	5	Orange.
396	325	4732 Ursæ Minoris	14 10 2	+69 57·0	5¼	Pale orange.
**397	326	α Böotis	14 10 39	+19 45·3	1	Golden yellow.
398		— Virginis	14 10 57	−16 0·8	8¾	Reddish.
399		Arg. +30 : 2513 Böotis	14 17 23	+29 52·9	6½	"Orange red" (Espin).
*400	327	4775 B.A.C. Böotis	14 18 53	+ 8 35·2	7	Deep orange. "Yellowish red" (Brodie). "? Col. var."
401	328	26342 Lal. Böotis	14 19 14	+26 12·2	8	Pale red. "Ruby red" (Brodie). "? Var."
402		106 Virginis	14 22 53	− 6 24·3	6	"Red" (*Uran. Arg.*).
403	329	— Virginis	14 23 55	− 5 29·5	8	Reddish. "Pale red" (Brodie). "In neb." (D'Arrest).
404	330	ρ Böotis	14 27 5	+30 51·2	3½	Full orange. "Reddish" (Brodie).
405	332	5 Ursæ Minoris	14 27 43	+76 11·2	4¼	Deep yellow.
406	334	4825 B.A.C. Böotis	14 30 8	+37 6·5	6½	Orange tinge. "Yellowish red" (Brodie).
407	335	α Centauri	14 32 7	−60 22·5	1	"Red." "Rich yellow" (Tupman).
408		— Libræ	14 34 31	−14 49·6	8	Red.
409		Arg. +33 : 2482 Böotis	14 34 37	+33 0·5	8¼	"Very red" (Espin).
410		Arg. +32 : 2504 Böotis	14 36 33	+32 1·5	8	"Red" (Espin).
411	337	34 Böotis	14 38 35	+26 59·8	5	Reddish orange.
**412	339	ε Böotis	14 40 11	+27 32·3	2½	Golden yellow.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
413	340	σ Libræ	14 43 50	−27 30·0	5	Red.
**414	341	β Ursæ Minoris	14 51 2	+74 36·3	2	Deep golden yellow. "? Var."
415		— Libræ	14 51 42	−11 59·4	7½	Red.
*416	342	4949 B.A.C. Ursæ Minoris	14 55 50	+66 22·2	4¾	Orange.
*417	343	20 Libræ	14 57 38	−24 50·9	3½	Orange red. "Reddish" (Bellamy). "Red" (Robinson). "Not red" (*Washburn Obs.*).
418	344	ν Libræ	15 0 29	−15 49·7	5½	Reddish orange.
419		4984 B.A.C. Libræ	15 3 26	−23 33·9	7½	Good red.
420	345	4976 B.A.C. Triang. Aust.	15 3 46	−69 39·8	6	"Almost scarlet" (J. Herschel). "Red" (Stone).
421	346	δ Lupi	15 11 8	−29 44·6	5	Red.
422	347	— Apodis	15 14 3	−75 32·0	7	"Very high red" (J. Herschel).
423		ϕ^1 Lupi	15 14 49	−35 51·7	3½	"Very red" (Gould). "Reddish yellow" (Williams).
424		6 Serpentis	15 15 24	+ 1 7·0	5½	"Red" (*Uran. Arg.*).
425		— Libræ	15 16 10	−28 47·2	8½	Perhaps reddish.
426	351	11 Ursæ Minoris	15 17 12	+72 13·4	5	Reddish.
427	352	ι Draconis	15 22 29	+59 21·1	3½	Yellow.
428		ϵ Trianguli Australis	15 26 39	−65 56·7	4½	"Orange" (Tupman).
429		11 Serpentis	15 27 18	0 48·8	6	"Red" (*Uran. Arg.*).
430	355	39 Libræ	15 30 21	−27 46·2	4	Decided red. "Not red" (*Washburn Obs.*).
431	356	r^1 Serpentis	15 31 22	+15 28·0	6½	Reddish. "Yellowish red" (Brodie). "? Var."
432		68 Espin Coronæ Borealis	15 33 31	+24 52·8	7	Red. "Orange ruby" (Webb).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
433	357	θ Ursæ Minoris	15 34 41	+77 42·9	5¼	Reddish.
434		κ Libræ	15 35 36	−19 19·3	5	Reddish orange.
435	358	κ Serpentis	15 43 47	+18 28·9	4	Orange.
436		ω Serpentis	15 44 44	+ 2 32·0	5	"Red" (*Uran. Arg.*).
437	361	— Coronæ Borealis	15 45 36	+39 54·4	8½	Good red. "Fine ruby" (J. Herschel). Var.
*438	363	ρ Serpentis	15 46 25	+21 18·6	4¾	Pale orange.
439	365	θ Libræ	15 47 33	−16 20·8	4¼	Pale orange.
440		28997 Lal. Libræ	15 50 57	−15 43·0	6¾	Red. "Reddish" (Robinson).
441	367	Arg. +47 : 2291 Coronæ Bor.	15 59 25	+47 32·3	7	Red.
442		6661 Lac. Normæ	15 59 42	−52 46·9	6¾	"Red" (Stone).
443		5347 B.A.C. Scorpii	16 1 25	−26 1·9	5¾	Good red.
444	369	W.B. XV. 1569 Herculis	16 2 37	+22 7·0	6½	Reddish. "Straw colour" (*Washburn Obs.*). "Pale yellowish red" (Brodie).
445		— Scorpii	16 3 3	−26 9·6	7½	Red. ? R.A.
446	370	47 Serpentis	16 3 10	+ 8 50·2	5¾	Decidedly reddish.
447	371	Arg. +9 : 3151 Serpentis	16 3 19	+ 8 55·2	7½	Orange.
448	372	29441 Lal. Serpentis	16 4 5	+ 1 6·8	7	Reddish. "Yellowish red" (Brodie). "Lemon yellow" (*Washburn Obs.*). "? Var."
**449	373	δ Ophiuchi	16 8 35	− 3 24·6	2¾	Reddish orange. "Orange red" (Robinson).
450	374	— Normæ	16 10 6	−45 32·0	8½	"Ruby red" (J. Herschel).
*451	375	ϵ Ophiuchi	16 12 29	− 4 25·3	3½	Golden yellow. "? Var."
**452	376	ν^1 Coronæ Borealis	16 18 13	+34 4·5	5	Orange.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
**453	377	ν^2 Coronæ Borealis	16 18 21	+33 57·4	5	Orange.
454	379	— Ophiuchi	16 20 37	−12 10·1	8	No star of this mag. here. "Red or crimson" (Brodie). "Dull brick red" (J. Herschel). "? Var." (Birm.). Dunèr's Var.
**455	381	α Scorpii	16 22 39	−26 11·2	1	Fiery red.
456	383	β Herculis	16 25 30	+21 43·8	2¾	Golden.
457		69 Espin Herculis	16 27 0	+35 27·6	6½	Reddish orange. =30129 Lal.
458	384	111 P. XVI. Scorpii	16 29 8	−35 1 7	5½	"Red."
459		— Ophiuchi	16 33 6	−12 6·3	8	Reddish.
460	385	— Scorpii	16 33 34	−32 9·7	8	"Deep red, like a drop of blood" (J. Herschel).
461		α Trianguli Australis	16 37 1	−68 49·5	2¼	"Orange; yellow" (Tupman).
462	386	Arg. +36 : 2772 Herculis	16 39 11	+36 43·2	8	Reddish; in the field with cl. 13 M.
463		η Aræ	16 40 17	−58 50·6	4	"Orange" (Tupman).
464		ϵ Scorpii	16 43 3	−34 5·5	2¼	"Orange; yellow" (Tupman).
465	388	30593 Lal. Ophiuchi	16 43 42	+ 0 7·0	8	Very slight tinge. ? Mag.
466	389	W.B. XVI. 1347 Herculis	16 43 48	+42 26·1	7	Pale orange.
467		ζ^2 Scorpii	16 46 50	−42 10·3	3	"Deep orange yellow" (Williams).
468		ζ Aræ	16 49 31	−55 48·9	4	"Bright orange" (Tupman).
469	394	30790 Lal. Ophiuchi	16 50 31	+ 1 35·9	8	Pinkish. "With red blush" (Brodie).
470	395	κ Ophiuchi	16 52 27	+ 9 32·8	3½	Golden. "Perhaps var."
471	399	30 Ophiuchi	16 55 16	− 4 3·5	5	Reddish orange. "Probably var."

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
472	400	61 Herculis	16 59 36	+34 33·6	7	Pale orange.
473		70 Espin Herculis	16 59 6	+35 33·8	6½	Reddish orange.
**474	402	α^1 Herculis	17 9 38	+14 31·0	3½	Deep orange. "Reddish" (Robinson). "Var."
475		— Ophiuchi	17 9 59	−15 5·3	7½	Fiery red.
476	404	π Herculis	17 11 13	+36 56·1	3½	Fiery orange. "? Var."
477	405	u Herculis	17 13 15	+33 13·0	5	Yellow.
478	407	— Ophiuchi	17 14 14	+ 2 16·2	7	Red.
479		73 Espin Herculis	17 15 50	+17 9·9	7½	Red. ? Decl.
480		β Aræ	17 16 9	−55 25·6	3	"Deep bright orange" (Tupman).
481	408	43 Ophiuchi	17 16 26	−28 2·3	5½	Reddish.
482		δ Aræ	17 21 10	−60 35·4	3	"Pale orange" (Tupman).
483	409	— Scorpii	17 22 49	−35 33·1	9	"Very deep red" (J. Herschel).
**484	410	— Ophiuchi	17 23 14	−19 23·1	8	Very decided red. "Ruby star" (J. Herschel). "Deep red, nearly crimson" (Brodie).
485	411	λ Herculis	17 25 17	+26 11·6	4¼	Pale orange.
486	412	β Draconis	17 27 57	+52 22·9	3	Golden.
487	414	— Scorpii	17 32 38	−41 33·5	8	"Ruby red" (J. Herschel).
488	415	— Aræ	17 33 51	−57 40·2	8	"Scarlet or high orange" (J. Herschel).
489	416	5986 B.A.C. Herculis	17 35 48	+31 15·7	6½	Yellowish.
490	417	β Ophiuchi	17 38 2	+ 4 36·8	3	Pale orange.
491	418	— Serpentis	17 38 28	−18 36·4	8½	Very little colour. "Probably var. in colour and mag."

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
492		— Ophiuchi	17 41 57	− 3 36·3	8	Reddish.
493		76 Espin Herculis	17 42 58	+36 7·4	6½	Reddish orange.
494		77 Espin Herculis	17 44 4	+36 35·6	6½	Reddish orange.
495		78 Espin Ophiuchi	17 45 36	+11 58·8	6¾	Orange. Assumed to be 877 Weisse.
496		79 Espin Ophiuchi	17 46 27	+ 1 8·1	7	Red. ? 886 Weisse.
497		80 Espin Ophiuchi	17 46 57	+ 1 20·4	6½	Fiery red.
498	421	ξ Draconis	17 51 35	+56 53·3	4	Pale orange.
499	422	— Ophiuchi	17 52 32	+ 2 44·0	7	Reddish. "? Var."
**500	423	γ Draconis	17 54 3	+51 30·1	2½	Fiery orange.
501	425	γ Sagittarii	17 58 44	−30 25·4	3	Reddish orange. "? Var."
502	427	— Sagittarii	18 3 24	−15 18·1	8	Slightly coloured. "Pale red" (Brodie). "? Var."
503	430	A (104) Herculis	18 7 47	+31 22·7	5	Orange.
504		7634 Lac. Sagittarii	18 8 3	−29 51·3	7½	"Reddish" (*Washburn Obs.*)
505	431	Arg. +22 : 3303 Herculis	18 8 15	+22 48·1	7½	Pale orange.
506		— Sagittarii	18 8 36	−18 56·9	8	Fiery red. ? Mag.
507	432	η Sagittarii	18 10 11	−36 47·8	3	"Deep golden yellow" (Schmidt).
508		— Sagittarii	18 11 55	−18 17·9	8	Reddish.
509		81 Espin Herculis	18 12 55	+17 55·5	7	Deep red.
510	433	Arg. +23 : 3299 Herculis	18 13 32	+23 14·3	7	Orange.
511	434	δ Sagittarii	18 13 57	−29 52·5	2¾	Orange red.
512	435	33933 Lal. Serpentis	18 13 49	+ 0 47·9	8	Reddish. "Very pale red" (Brodie).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
*513	437	33896 Lal. Herculis	18 16 57	+25 0·2	7½	Rich orange. "No red star here" (*Washburn Obs.*). ? R.A. too great by 1ᵐ 28ˢ; Decl. too great by 35′.
514	438	— Serpentis	18 18 17	− 1 38·3	7	Yellowish. "Double: A, 6, yellow; B, 7, blue" (Secchi).
515	439	21 Sagittarii	18 18 46	−20 36·1	5	Red.
*516	440	109 Herculis	18 18 59	+21 43·5	4	Golden yellow.
517	441	λ Sagittarii	18 21 11	−25 28·9	3	Reddish orange.
518		60 Serpentis	18 23 57	− 2 3·4	5½	"Red" (*Uran. Arg.*)
**519	446	6306 B.A.C. Sagittarii	18 26 27	−14 56·4	5¾	Fiery red. "Golden yellow" (Brodie). "Reddish" (Bellamy).
520	447	34307 Lal. Aquilæ	18 27 16	− 5 14·5	7½	Tinge of orange. "? Colour var." "Very pale red" (Brodie).
**521	448	Arg. +36 : 3168 Lyræ	18 28 30	+36 54·5	8	Decided crimson. "? Var."
*522	449	1 Aquilæ	18 29 13	− 8 18·9	4	Good orange.
523	451	123 P. XVIII. Clyp. Sob.	18 30 12	− 6 49·9	7	Reddish.
524		82 Espin Lyræ	18 30 23	+38 21·1	7	Red.
525	452	6341 B.A.C. Herculis	18 30 57	+23 30·0	5¾	Orange.
526	453	34507 Lal. Aquilæ	18 32 40	−13 52·6	8	Scarcely tinged. Colour changed since 1873.
527	454	— Serpentis	18 32 42	+11 21·3	8½	Reddish.
528		83 Espin Lyræ	18 33 46	+37 41·0	7	Red. "Pale ruby" (Webb).
529	457	34746 Lal. Aquilæ	18 38 22	− 6 38·9	7	Reddish.
530	458	Arg. +36 : 3243 Lyræ	18 39 1	+36 51·2	8	Fiery red. ? Var. "No really red star here" (*Washburn Obs.*).
531	459	Arg. +39 : 3505 Lyræ	18 39 37	+39 11·5	7	Slightly red. ? Colour var. Has a small *comes*.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
532	461	Arg. +33 : 3192 Lyræ	18 40 50	+33 4·5	8	Reddish.
533	463	Arg. +18 : 3817 Herculis	18 41 52	+18 35·1	7	Pale orange.
**534	464	— Clypei Sobieskii	18 43 50	− 8 1·9	8	Rich fiery red. "? Var." "Orange" (Brodie).
*535	466	ν^1 Sagittarii	18 47 32	−22 52·8	5	Red.
536	468	ν^2 Sagittarii	18 48 27	−22 48·5	5¼	Pale orange.
537		62 Serpentis	18 50 6	− 6 28·8	6	"Red" (*Uran. Arg.*).
538	470	δ^2 Lyræ	18 50 39	+36 45·5	4½	Fiery orange.
539		9 Aquilæ	18 51 10	− 5 59·3	5	"Red" (*Uran. Arg.*).
*540	471	ξ^2 Sagittarii	18 51 9	−21 15·1	3½	Orange.
541	472	W. B. XVIII. 1528 Herculis	18 51 15	+17 58·3	5¾	Pale orange.
542	473	— Aquilæ	18 51 57	+ 0 18·5	9	Red. "Orange red" (Brodie).
*543	475	— Aquilæ	18 52 31	+14 12·6	9	Deep red.
544		87 Espin Lyræ	18 53 16	+36 19·3	7	Orange.
545	476	Arg. +38 : 3362 Lyræ	18 53 23	+38 39·2	7½	Pale orange.
546	477	Arg. +22 : 3549 Vulpeculæ	18 55 19	+22 39·7	6½	Golden.
**547	478	12 Aquilæ	18 55 47	− 5 53·5	4	Orange. "? Var."
548	479	λ Lyræ	18 55 51	+31 59·4	5¼	Orange. "? Var."
549	480	35562 Lal. Aquilæ	18 57 4	+ 8 12·8	6½	Pale orange.
550	481	*o* Sagittarii	18 58 5	−21 54·1	4	Full yellow.
551	482	35624 Lal. Aquilæ	18 58 15	+ 8 8·1	8	Decided yellow.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
**552	483	35611 Lal. Aquilæ	18 58 32	− 5 50·9	7½	Very fine fiery red. "? Var." "Decided red" (Brodie). "Deep red" (Wickham).
553	485	τ Sagittarii	19 0 5	−27 49·6	3½	Reddish orange.
554	487	35928 Lal. Vulpeculæ	19 4 2	+24 0·3	7	Reddish.
555		89 Espin Lyræ	19 4 28	+38 58·7	7	Orange. =78 Weisse.
556	490	Arg. +18 : 4011 Sagittæ	19 10 42	+18 19·8	7	Orange.
557	491	δ Draconis	19 12 32	+67 28·0	3¼	Fine golden yellow.
558		91 Espin Lyræ	19 13 22	+38 47·6	6¾	Red.
559	492	— Lyræ	19 14 49	+27 3·2	9	Red. "? Var."
560	493	Arg. +22 : 3660 Vulpeculæ	19 14 51	+22 21·9	8	Reddish.
561		93 Espin Sagittæ	19 16 43	+17 27·5	9½	Good red. "? Var." (Espin).
562		4 Vulpeculæ	19 20 38	+19 35·0	5¼	Orange; fine field.
*563	494	3 Cygni	19 20 54	+24 42·9	6	Orange.
**564	495	— Vulpeculæ	19 21 30	+19 34·8	6	Yellowish.
565	496	128 P. XIX. Vulpeculæ	19 21 40	+19 40·3	6¼	Reddish orange.
*566		94 Espin Cygni	19 21 44	+50 1·0	7	Fiery red.
567	498	Arg. +1 : 4004 Aquilæ	19 22 17	+ 1 57·2	8	Reddish. "? var."
568	499	Arg. +2 : 3904 Aquilæ	19 24 39	+ 2 40·6	6½	Reddish orange.
*569	500	36 (*e*) Aquilæ	19 24 54	− 3 1·1	5¼	Strong reddish orange. "Light orange" (Brodie).
570		179 Espin Cygni	19 25 21	+45 48·9	8	"Very red; ? var." (Espin).
571	501	Arg. +1 : 4021 Aquilæ	19 25 30	+ 1 47·4	8	Golden.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
**572	502	6702 B.A.C. Draconis	19 25 30	+76 21·1	6½	Strong fiery red. "? Var." "Reddish orange" (Brodie).
**573	503	β Cygni	19 26 17	+27 43·7	3	Rich yellow, with a 7th mag. *comes* blue. "? Var."
574	504	Arg. +4 : 4152 Aquilæ	19 27 43	+ 4 47·6	7	Reddish orange.
575	505	36981 Lal. Sagittarii	19 28 0	−16 37·8	7½	Orange. "Good ruby" (J. Herschel). "Fiery red" (Gore, 1876). "Red; deeper at times" (Bellamy). ? Var. in colour; no trace of ruby (G.F.C.).
576	506	Arg. +5 : 4190 Aquilæ	19 28 23	+ 5 13·5	7	Golden.
577		97 Espin Vulpeculæ	19 31 12	+23 36·3	8	Reddish.
578		98 Espin Vulpeculæ	19 35 13	+17 38·4	8	Rich orange. "Pale ruby" (Webb).
579	509	Arg. +32 : 3522 Cygni	19 36 55	+32 21·9	8	Red.
580	510	37504 Lal. Aquilæ	19 38 44	+ 4 42·0	8	Reddish. "Reddish tinge" (Brodie). "? Var."
581	511	Arg. +12 : 4060 Aquilæ	19 39 29	+12 58·0	7	Reddish.
*582		101 Espin Cygni	19 40 16	+40 26·6	6	Fiery red.
583	512	γ Aquilæ	19 41 2	+10 20·7	2¾	Brilliant orange. "Reddish yellow" (Robinson).
584		104 Espin Aquilæ	19 42 18	+ 9 45·9	8	Red, with blue companion; dist. ± 50″.
585	515	Arg. +22 : 3812 Vulpeculæ	19 43 28	+22 29·9	8	Reddish.
586	518	χ Cygni	19 46 20	+32 38·3	Var. 4–0	Fiery red, when approaching max.
587	519	19 Cygni	19 46 39	+38 26·1	5½	Full orange.
588	521	Arg. +43 : 3425 Cygni	19 53 37	+43 57·5	8	Fiery red.
589		109 Espin Sagittæ	19 55 8	+17 18·3	7½	Fine orange; there is a red 9th mag. S.
590	522	*c* Sagittarii	19 55 53	−28 1·0	4¾	

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
591		181 Espin Cygni	19 56 30	+30 31·2	9¼	"Very red" (Espin).
592	525	Arg. +36 : 3820 Cygni	19 57 13	+36 47·6	7	Pale orange.
593		38428 Lal. Cygni	19 59 41	+38 0·6		Reddish.
594	526	Ö-Arg. (2) 20234 Sagittarii	20 0 12	−27 32·5	7	"Ruby" (J. Herschel). "Deep orange red" (Brodie).
595	528	— Pavonis	20 1 38	−60 15·2	8½	"Very red" (J. Herschel).
596	530	Arg. +16 : 4153 Sagittæ	20 3 8	+16 21·4	6½	Orange.
597	537	19 Vulpeculæ	20 7 12	+26 29 0	5¾	Reddish orange.
*598	538	66 Aquilæ	20 7 33	− 1 20·3	5¾	Orange.
599	541	Arg. +38 : 3957 Cygni	20 9 24	+38 23·7	8	Decided red.
*600	543	o^1 Cygni	20 10 10	+46 24·5	3¾	Fine golden yellow; *comes* blue.
601	544	Arg. +36 : 3956 Cygni	20 10 25	+36 19·6	8	Reddish.
*602	545	Ö-Arg. (2) 20363 Capricorni.	20 10 40	−21 39·3	7½	Decided red. "Perhaps the finest of my ruby stars" (J. Herschel). "Deep ruby; fine colour" (Brodie). "Bright ruby" (Bellamy).
603	546	23 Vulpeculæ	20 11 13	+27 28·6	4¾	Full orange.
**604	547	α^1 Capricorni	20 11 33	−12 50·9	4½	Golden yellow.
**605	548	α^2 Capricorni	20 11 57	−12 53·1	3¾	Golden yellow.
606	549	o^2 (32) Cygni	20 12 5	+47 22·6	4½	Pale orange.
607	551	6986 B.A.C. Cygni	20 12 59	+40 1·3	5½	Reddish orange.
*608	550	σ Capricorni	20 13 3	−19 27·7	5½	Double. A golden; B colourless or ruddy. Dist. 50″.
609	552	38988 Lal. Aquilæ	20 13 21	+ 0 15·2	8¾	Red.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
610	553	U Cygni	20 16 12	+47 32·9	7–11	Red. "Remarkable ruby." "Var."
611		115 Espin Cygni	20 17 37	+53 14·2	7	Reddish.
612	555	7027 B.A.C. Cygni	20 18 50	+40 40·4	6	Reddish yellow.
613	556	— Aquilæ	20 19 10	+ 0 11·8	10	Reddish. ? Var. Mag. 8½, Aug. 9, 1885.
614	557	39 Cygni	20 19 28	+31 50·1	4½	Reddish orange.
615	558	39304 Lal. Delphini	20 20 27	+ 9 42·0	7	Pale orange. "Probably var."
616		116 Espin Cygni	20 20 46	+39 47·5	7½	Reddish. "Orange ruby" (Webb).
617	559	— Capricorni	20 21 12	−28 37·4	8	Decided red. "Fine ruby" (J. Herschel).
618	560	ω^1 Cygni	20 23 40	+49 1·1	5	Golden yellow.
*619		117 Espin Delphini	20 24 2	+15 53·8	8½	Good red. There is a blue 8th mag. S.
620		118 Espin Cygni	20 24 47	+39 36·7	9	Fiery red. "Colour very fine" (Espin).
*621	562	ω^3 Cygni	20 27 55	+48 50·9	5½	Deep golden yellow.
622	563	47 Cygni	20 29 37	+34 52·4	4¾	Orange.
*623	564	70 Aquilæ	20 31 0	− 2 55·9	5¼	Full orange.
624	565	— Aquilæ	20 32 41	+ 0 37·6	8½	Reddish.
*625	566	Arg. +17 : 4370 Delphini	20 32 54	+17 52·7	7	Full orange.
626	569	Arg. +17 : 4401 Delphini	20 40 25	+17 41·5	7	Reddish. "Probably var."
**627	570	ϵ Cygni	20 41 45	+33 33·4	2¾	Fine golden yellow.
628		119 Espin Cygni	20 43 4	+45 38·7	8½	Pale ruby.
629		120 Espin Cygni	20 45 4	+45 26·7	8½	Fiery red.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
630		121 Espin Cygni	20 46 8	+50 22·0	7½	Red.
631		122 Espin Cygni	20 47 20	+49 42·6	7	Deep orange.
632	572	Arg. +32 : 3980 Cygni	20 49 26	+33 0·0	5¾	Orange.
633	573	1294 Bess. Delphini	20 52 4	+15 49·8	8	Yellowish. "Pale orange tinge" (Brodie). "Probably var. both in mag. and colour."
634		123 Espin Cygni	20 54 10	+45 48·9	8	Fiery red.
635		ζ (3) Equulei	20 59 6	+ 5 3·9	5¾	Fiery red. "Red" (*Uran. Arg.*)
636	575	A Capricorni	21 0 42	−25 26·7	4½	Deep orange.
637	576	ξ Cygni	21 0 56	+43 29·3	3¾	Deep golden yellow.
638		— Capricorni	21 1 6	−16 51·7	8½	Reddish.
639		124 Espin Cygni	21 6 40	+47 12·4	7½	Orange; a blue 7¼ mag. to the N.
*640	579	61 P. XXI. Cephei	21 10 2	+59 39·7	7½	Remarkable fiery red. "Orange red" (Brodie). "? Var."
641	580	8745 Lac. Indi	21 14 19	−70 11·7	6	"Red, inclining to orange" (J. Herschel).
642		125 Espin Cygni	21 14 58	+49 36·1	7	Fiery red.
*643	582	— Cygni	21 17 53	+41 55·7	9	Deep red.
644		33 Capricorni	21 17 55	−21 19·1	5½	"Red" (Gore).
645		184 Espin Cygni	21 19 5	+40 28·1	7½	"Red" (Espin).
*646	583	2 Pegasi	21 24 58	+23 9·4	4¾	Bright orange.
647	585	Arg. +45 : 3584 Cygni	21 29 9	+45 21·9	7	Reddish orange.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
648	586	— Cephei	21 30 37	+58 13·4	6	Intense orange.
649	587	Arg. +44 : 3877 Cygni	21 31 53	+44 53·1	7	Reddish orange.
650	589	889 Bess. Cygni	21 37 23	+35 0·4	7	Unmistakable fiery red.
651	590	— Cygni	21 37 24	+42 20·4	7	Strong reddish tinge. "Schmidt's *Nova* of 1876."
652		129 Espin Cygni	21 37 53	+48 41·5	8¾	Orange.
**653	591	ϵ Pegasi	21 38 47	+ 9 22·2	2½	Full orange. "Probably var."
654		130 Espin Cephei	21 38 41	+70 17·1	8	Orange.
**655	592	923 Bess. Cygni	21 38 43	+37 30·7	8	Deep fiery red or pale crimson. "Good red" (Brodie).
**656	594	μ Cephei	21 40 8	+58 16·5	4-6	Intense reddish orange. "Light orange" (Brodie).
657		47 Capricorni	21 40 24	− 9 47·0	6½	"Orange red" (Espin).
**658	596	42431 Lal. Aquarii	21 40 50	− 2 43·4	6½	Decided red. "? Var." "Red" (Robinson).
659		131 Espin Cephei	21 42 45	+57 33·4	8	Reddish orange.
660	598	— Cygni	21 51 6	+49 59·3	9	Pale red.
*661	599	7658 B.A.C. Cephei	21 53 33	+63 5·7	5¼	Fiery orange.
*662		133 Espin Cephei	21 54 22	+65 37·6	6½	Fiery red. Blue star 6½ mag. near.
*663	600	— Pegasi	21 58 59	+27 49·1	7½	Fiery red. "Orange tinge" (Brodie). "? Var."
664	601	18 Cephei	22 0 39	+62 33·8	5½	Reddish orange.
665	602	20 Cephei	22 1 40	+62 14·9	5½	Orange.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
666		134 Espin Lacertæ	22 6 30	+39 9·9	7¼	Reddish.
667	604	ζ Cephei	22 7 3	+57 39·5	3½	Pale orange.
668	606	7766 B.A.C. Cephei	22 8 56	+62 43·5	6	Reddish orange.
669	607	7765 B.A.C. Lacertæ	22 9 9	+39 10·0	4½	Pale orange.
670	608	1 Lacertæ	22 11 10	+37 12·1	4¼	Bright yellow.
671	609	43501 Lal. Pegasi	22 11 56	+ 4 35·8	7¾	Dull pale red. "Tinge of red" (Brodie).
672	610	7813 B.A.C. Cephei	22 18 59	+55 24·4	7	Good red. "Orange" (Brodie).
673	611	β Lacertæ	22 19 14	+51 40·8	4½	Golden.
674		35 Pegasi	22 22 17	+ 4 8·7	5	Deep orange. "Red" (*Uran. Arg.*).
675		36 Pegasi	22 23 39	+ 8 34·1	6	Reddish orange. "Red" (*Uran. Arg.*).
676	612	5 Lacertæ	22 24 57	+47 8·7	4½	Reddish orange.
677	613	δ Cephei	22 25 5	+57 51·1	3½–4½	Orange; with blue companion in good contrast. "Var."
678		β Piscis Australis	22 25 15	−32 54·6	5	"Wide double. A white; B 8, reddish lilac" (Gore).
679		135 Espin Cephei	22 26 11	+56 25·8	8	Fiery red.
680	615	Arg. +56 : 2821 Cephei	22 34 18	+56 13·6	5½	Reddish.
681		136 Espin Cephei	22 30 23	+57 36·1	7½	Fiery red.
682		137 Espin Cephei	22 32 35	+57 51·4	7¼	Red. "Var. 7–8" (Espin).
683	616	11 Lacertæ	22 35 40	+43 42·1	4¾	Reddish.
684	617	β Gruis	22 36 6	−47 27·7	3	"Reddish." "Orange" (Williams).

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
685	618	g^2 Aquarii	22 41 39	−20 31·0	5½	Golden yellow.
*686	620	τ^2 Aquarii	22 44 47	−14 10·4	4	Orange. "? Var."
687	621	λ Aquarii	22 46 52	− 8 9·9	3¾	Full golden yellow. "Reddish" (Robinson).
688	622	15 Lacertæ	22 47 5	+42 43·7	5	Reddish.
689		138 Espin Lacertæ	22 52 25	+42 25·1	7	Orange.
690	625	267 P. XXII. Aquarii	22 53 7	−25 45·0	6	Reddish. "Pale red" (Brodie).
691	626	— Piscium	22 55 39	+ 0 29·6	8½	Slightly tinged. "Reddish tinge" (Brodie).
**692	627	β Pegasi	22 58 25	+27 29·0	2½	Rich golden.
**693	629	55 Pegasi	23 1 27	+ 8 48·8	4½	Deep yellow.
694	630	57 Pegasi	23 3 59	+ 8 4·8	5½	Reddish orange.
695		W.B. XXIII. 48 Piscium	23 5 38	+ 4 24·2	8	Orange.
696	631	Aquarii 286	23 8 2	−13 59·8	7	Reddish. "Double, A 7; B 10; dist. 1″·40."
697	632	φ Aquarii	23 8 37	− 6 38·3	4¼	Deep orange.
698	633	ψ^1 Aquarii	23 10 5	− 9 41·1	4½	Orange.
699	634	χ Aquarii	23 11 9	− 8 19·4	5¼	Orange.
**700	635	8 Andromedæ	23 12 38	+48 24·9	5	Fine golden yellow. "Orange red" (Brodie).
701	636	94 Aquarii	23 13 28	−14 3·4	5¼	Orange.
702	637	262 Bess. Pegasi	23 14 44	+22 29·4	7	Orange. "? Var."
703	639	2557 Arg. Cassiopeiæ	23 19 22	+60 59·6	8¾	Reddish. In cl. h 2238.

Number. G. F. C.	Birm.	Star.	R.A. 1890. h m s	Dec. 1890. ° ′	Mag.	Colour; Remarks.
704	642	46112 Lal. Pegasi	23 27 0	+23 14·3	7	Pale orange.
705	643	71 Pegasi	23 27 59	+21 54·9	5½	Deep orange.
706	644	λ Andromedæ	23 32 10	+45 51·7	4	Golden yellow.
707	645	77 Pegasi	23 37 46	+ 9 43·2	5½	Deep orange.
708	647	78 Pegasi	23 38 27	+28 45·1	5	Pale orange.
**709	648	19 Piscium	23 40 45	+ 2 52·5	5¼	Decided red. "Probably var."
710	651	4154 Groom. Cephei	23 47 0	+74 55·0	6¼	Pale orange.
711	652	ρ Cassiopeiæ	23 48 53	+56 53·2	4½	Yellow.
712	653	235 P. XXIII. Pegasi	23 51 5	+22 2·1	6	Reddish. "Slightly red" (*Uran. Oxon.*).
713	654	9659 Lac. Sculptoris	23 51 28	−27 14·2	6½	Reddish.
**714	655	ψ Pegasi	23 52 9	+24 31·9	4½	Reddish yellow.
**715	656	R Cassiopeiæ	23 52 49	+50 46·4	5–12	Very red. "Vivid red" (Brodie). "Var."
716	657	W.B. XXIII. 1090 Piscium	23 54 56	+ 0 27·1	8¾	Pale orange.
**717	658	6259 Rad. Cassiopeiæ	23 55 39	+59 44·5	8	Fiery red. A 9th mag. blue star near. "Orange red" (Brodie).
*718		30 Piscium	23 56 19	− 6 37·5	4½	Fiery red. "Red" (*Uran. Arg.*).
719		33 Piscium	23 59 42	− 6 19·4	4¾	Orange. "Red" (*Uran. Arg.*).

Northfield Observatory, Eastbourne, Sussex:
April, 1887.

Description of a New Measuring Rod. By Edward Crossley.

Some ten years ago I had occasion to triangulate across a valley 3,000 feet wide on the south side of my observatory at Bermerside, Halifax. In order to accomplish this with sufficient accuracy it was necessary to measure a base line to 1 in 10,000. I soon discovered that no *simple* and *accurate* method had yet been devised for this purpose. The ordinary surveyor's chain or steel tape was quite inadequate for the purpose, especially as the ground was more or less uneven. The elaborate methods used in geodetic surveys were also out of the question on account of their great complexity. I therefore determined to construct a measuring rod on simple geometric principles, capable of doing accurate and rapid work over uneven ground, with or without the aid of an assistant.

The first rod I designed for this purpose was made for me by Messrs. T. Cooke and Sons of York. It was made of well-seasoned red deal, 1 inch thick, 6 feet in length, with a T section $2\frac{1}{2}$ inches wide and 4 inches deep in the centre. At each end, on the under flat surfaces, brass plates were attached with spherical cups in their centres, facing downwards. In the middle of the rod was placed a clinometer reading to 30″ of arc. Three stout, firm tripods were used for carrying the rod. Each tripod terminated in a flat iron ring, faced in the lathe, $6\frac{1}{2}$ inches in outside diameter and 4 inches internal diameter. On these rings were placed brass pedestals with circular bases 6 inches diameter, surmounted by spheres or balls $\frac{7}{8}$ inch diameter, the top of the balls not being more than $2\frac{1}{2}$ inches above the surface of the rings. Each pedestal had a spindle going through the ring, with a brass tribrach "washer" and clamp, milled head screw nut on the under side of the iron ring, so that the pedestal could be clamped anywhere within the area of the ring. The spherical heads were turned of equal size, to fit the cups on the measuring rod. In the first instance stout posts with brass-headed nails in their centres were used for terminals.

The method of procedure was as follows. A tripod was set up over the first post and the spherical head clamped, with the plumb bob over the brass nail. A correction was applied for any inclination of the ring. A piece of string 60 feet in length with tabs every 6 feet was stretched on the ground in the direction of the base line, to facilitate the setting of the tripods. B tripod was set over the first tab and C tripod over the second, with the heads unclamped. The measuring rod was now placed upon the heads of A and B tripods. The head of B tripod being unclamped was free to take the position necessary to fit the cup at the end of the rod.

After lining, B head was clamped and the clinometer read off. Then the same process was repeated on B and C tripods.

A tripod was set forward over the tab in advance of C, and so on to the end of the base.

The following results were thus obtained in 1883 from three measurements of the same base line :—

	Feet.	Differences.		
			μ	in.
I.	442·635	15	34	$\frac{1}{5}$
II.	442·612	8	18	$\frac{1}{10}$
III.	442·612	8	18	$\frac{1}{10}$
	442·620	10	23	$\frac{1}{8}$

N.B.—μ = millionths of the whole distance.

These results were so encouraging that I determined to go a step further, and secure results of an accuracy of not less than 1 in 100,000.

For this purpose I made another rod, 10 feet in length, of well-seasoned pine, with a T section, but with an opening $\frac{1}{2}$ inch square from end to end, through which a steel rod $\frac{1}{4}$ inch diameter passed. The steel rod was firmly attached at each end to brass plates with spherical cups, and two thermometers were also attached to the rod. Fresh terminals were used, consisting of iron castings, with $\frac{1}{2}$-inch vertical holes in their centres, buried in the ground, the surface of the ground being made good by iron box frames and lids.

Adopting precisely the same method as before, the following results were obtained in 1886 :—

	Feet.	Differences.			Feet.	Differences.		
			μ	in.			μ	in.
I.	836·33022	461	5·5	$\frac{1}{18}$	442·89724	72	1·6	$\frac{1}{111}$
II.	836·33786	303	3·6	$\frac{1}{28}$	442·89682	114	2·3	$\frac{1}{71}$
III.	836·33640	157	1·9	$\frac{1}{52}$	442·89981	185	4·2	$\frac{1}{45}$
	836·33483	307	3·7	$\frac{1}{27}$	442·89796	124	2·8	$\frac{1}{66}$

Thus a greater accuracy than 1 in 100,000 was obtained.

The 10-feet measuring rod was afterwards further improved by making the wood portion of square box section, and supporting the steel rod on rollers along the centre of the box. Simple means were also adopted for measuring intermediate lengths with an accuracy of 1-100th of an inch. The thermometers were bedded to the steel rod with mercury amalgam. A detached thermometer was also placed inside the box near to one of the other thermometers, and the steel rod was blackened throughout its whole length to increase its thermal conductivity, the weight of the whole rod being about 12 lbs.

In order to determine the exact value of the measuring rod I have designed a variable end standard, which Messrs. Troughton and Simms have constructed for me.

It consists of a cold-drawn steel tube $1\frac{1}{4}$ inch external diameter, with a telescopic or sliding piece at one end having a range of 1 inch. The ends are parallel plane surfaces. By means of a vernier, scale, and reading microscope, the position of the telescopic end in reference to the tube is easily determined by estimation to 0·0005 inch, and with care to 0·00025, the reading being to 0·001.

The tube is supported in a stout mahogany box at two points, on a roller at one, and suspended by a vertical arm and cross-piece at the other. The box is provided with brass brackets at each end, by means of which it rests upon the spherical heads. One bracket has a spherical cup, the other bracket a cylindrical groove.

By means of suitable screws and springs the plane ends of the steel tube are brought into firm contact with the spherical heads, so that the centres of the spheres lie in the axis of the tube. The temperature of the tube is indicated by two thermometers.

Most of the comparisons have been taken on two iron brackets firmly attached to the stone wall of the observatory, but I have constructed a wooden bench carrying two iron plates at each end, connected by two steel rods from end to end in such a manner as to withstand the springs of the steel tube or standard 10-feet measure. In this way I have got results about 0·0002 inch less than when the comparisons were made on the iron brackets.

Six sets of comparisons, consisting of five each set on the iron brackets, gave 10·00296 feet as the value of the measuring rod, with a mean error of each set of 0·000015 foot. Another six sets on different days gave 10·002951 feet as the value, with a mean error of each set of 0·000012 foot. With greater reading power I have no doubt that one set of comparisons would give the value of the rod to 0·0001 inch or 0·00001 foot.

The average difference of the two thermometers on the 10-feet standard in 125 comparisons was 0°·93 F., and the 10-feet measuring rod under similar circumstances 0°·69 F. The difference between the attached and detached thermometers adjoining one another on the measuring rod in 145 comparisons was 0°·16 F., showing a close approximation of the steel rod to the temperature of the air. Two persons can make five comparisons in $12\frac{1}{2}$ minutes.

Experiments for expansion gave a coefficient of 61·5 ten-millionths for 1° F. for both rods, with a probable error of 1·0 ten-millionth. One hundred feet have been set off with the measuring rod eight times with a mean error of 0·0005 foot, showing that the apparatus is capable of doing reliable work to 1 in 200,000. One hundred yards can be measured easily in one hour with one assistant.

The accompanying drawings will make the above description clear.

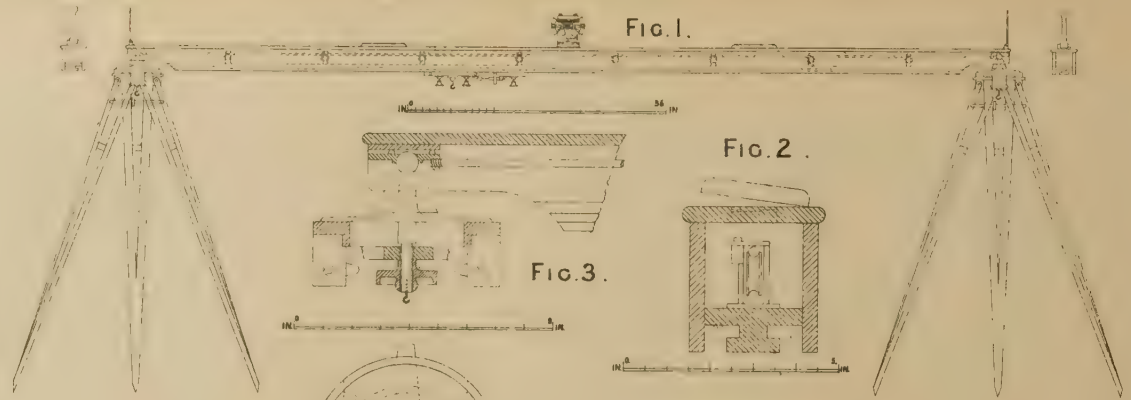


Fig. 1.

Fig. 3.

Fig. 2.

Fig. 4.

Fig. 6.

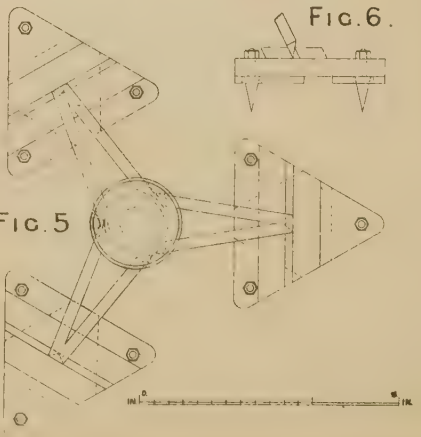
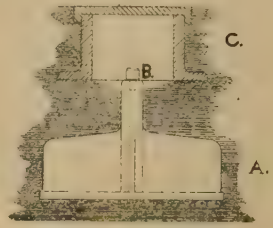


Fig. 5

Fig. 8.

Fig. 7.



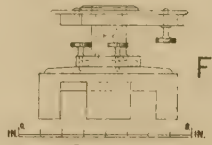


Fig. 10.

Fig. 9.

Fig. 11.

Fig. 1.

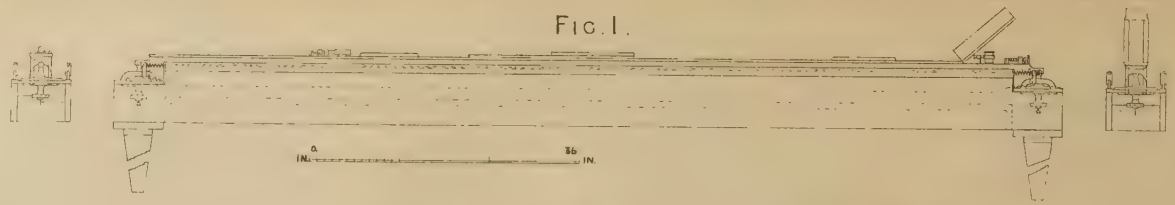


Fig. 2.

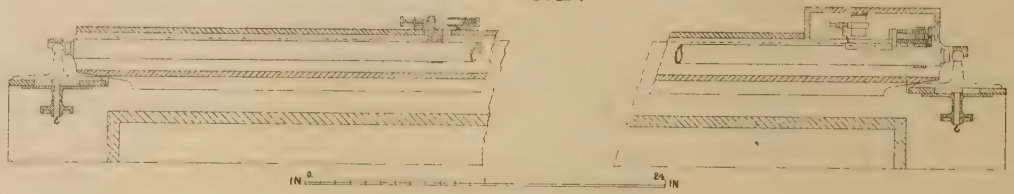


Fig. 9.

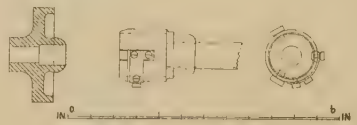
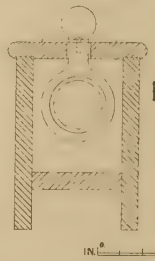


Fig. 3.



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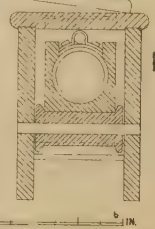

Fig. 5.

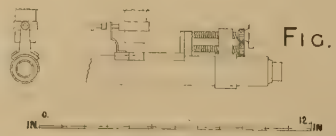

Fig. 6.

Fig. 7.

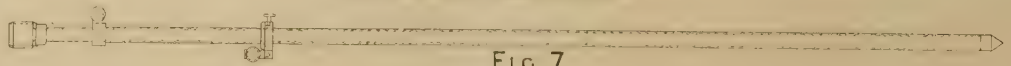

Fig. 8.

Plate 1.

Fig. 1 is the measuring rod of square box section resting on two tripods.

Fig. 2 is a section of the rod. The steel rod is indicated in both drawings passing through the middle of the box on rollers. At each end of the rod are sights for lining. In the centre is the clinometer, and between these the two thermometers.

Underneath the rod is a sliding piece, with clamps and slow motion and hook for plumb bob for measuring part lengths, the rod being divided every six inches, an ivory scale divided on both edges giving intermediate lengths to the hundredth of an inch.

Fig. 3 shows the end of the rod and the top of the tripod in section, and fig. 4 the top of the tripod in plan.

Figs. 5 and 6 show a method of supporting the tripods so as to facilitate the levelling of the rings, by moving the feet along the grooves tangentially, and thus tilting the rings to any desired position.

Fig. 7 shows the terminal mark, consisting of an iron casting A, surmounted by a brass step with cap at B. C is the box frame and lid for making good the surface of the ground.

Fig. 8 is a plan of C.

Fig. 9 is a plan of A.

Figs. 10 and 11 show a pedestal suitable for supporting a theodolite centrally over the spherical head on one of the tripods.

Plate 2.

Fig. 1 shows the standard 10-feet rod resting upon two spherical heads upon the wooden comparing bench.

Fig. 2 shows the above in longitudinal section.

Figs. 3 and 4 are sections of the rod.

Figs. 5 and 6 show the telescopic end of the steel tube, with reading microscope and adjusting screws.

Figs. 7 and 8 show a telescopic tubular rod, which is more convenient than the plumb bob. The lower end is conical, and fits into the brass step on the terminal. The upper end is cylindrical, and embraces the spherical portion of the milled head as shown in section, fig. 9. The rod will then rotate on these two centres, and by means of the attached level the vertical position is easily found with considerable accuracy.

Observations of Comet b 1887 (*Brooks*) *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The observations were made with the East or Sheepshanks Equatorial, aperture 6·7 inches, by taking transits over two cross wires at right angles to each other, and each inclined 45° to the parallel of Declination.

1887 *Comet b* (*Brooks*).

Greenwich Mean Solar Time.	Observer.	✱−★ R.A.	Corr. for Par. and Refraction in R.A.	✱−★ N.P.D.	Corr for Par. and Refraction in N.P.D.	No. of Comp.	Apparent R.A.	Apparent N.P.D.	Comp. Star.
1887 h m s		m s	s	′ ″	″		h m s	° ′ ″	
March 13 8 12 24	A. D.	−2 1·17	+0·30	+ 9 9·7	−1·7	3			*a*
13 8 30 1		−2 36·50	+0·30	− 0 31·6	−1·9	3	3 57 48·90	45 37 34·7	*b*
16 11 47 59	H. T.	−0 44·64	+0·30	+ 5 24·3	−3·9	5	4 4 4·53	48 15 50·6	*c*
18 8 17 5	H. T.	+0 22·12	+0·20	− 3 43·9	−2·1	6	4 7 33·75	49 44 29·1	*d*
23 8 0 51	H.	+1 20·66	+0·20	+ 4 35·8	−2·1	5	4 16 17·28	53 26 25·9	*e*
23 8 35 56		−1 48·70	+0·20	− 8 25·7	−2·4	3	4 16 20·71	53 27 15·9	*f*
24 9 58 27	T.	−0 23·31	+0 30	− 1 22·9	−3·0	8			*g*
24 10 11 34		+2 22·50	+0·30	− 2 7·8	−3·1	1			*h*
24 10 11 34		+2 1·50	+0·30	−10 2·8	−3·1	1	4 18 11·32	54 11 45·0	*i*
27 10 11 3	H. T.	−0 31 37	+0·30	− 1 40·9	−3·2	6			*j*

Mean Places of Comparison Stars.

	Star's Name.	R.A. 1887·0	N.P.D. 1887·0	Authority.
		h m s	° ′ ″	
a	Arg. Zone +44° 859	3 59 47	45 29	Bonn Observations, Vol. V.
b	W. B. (2) III. 1241	4 0 25·52	45 38 8·5	Weisse's Bessel (2)
c	W. B. (2) IV. 8	4 4 49·29	48 10 29 0	Weisse's Bessel (2)
d	*f* Persei	4 7 11·88	49 48 13·5	Greenwich Observations, 1886
e	W. B. (2) IV. 259	4 14 56·92	53 21 49·2	Weisse's Bessel (2)
f	W. B. (2) IV. 337	4 18 9·69	53 35 41·3	Weisse's Bessel (2)
g	Arg. Zone +35° 858	4 15 44	54 13	Bonn Observations, Vol. IV.
h	W. B. (2) IV. 287	4 16 10·03	54 21 48·1	Weisse's Bessel (2)
i	Arg. Zone +35° 865	4 18 27	54 12	Bonn Observations, Vol. IV.
j	Arg. Zone +33° 871	4 23 23	56 14	Bonn Observations, Vol. IV.

The observations are corrected for parallax and refraction. The initials H. T., A. D., T., H. are those of Mr. Turner, Mr. Downing, Mr. Thackeray, and Mr. Hollis respectively.

Lunar Occultations on March 29, 1887. By C. Leeson Prince.

I observed the occultations of five stars this evening under very favourable conditions.

The grey light of the non-illuminated portion of the Moon was remarkably distinct at the limb, and its gradual approach to each star could not be watched with greater exactness.

	Local Sidereal Time. Disapp.	Reapp.
	h m s	h m s
θ^1 Tauri	9 46 17·5	10 42 6
θ^2 „	9 55 22	10 36 56
A small star	10 0 47·5	
A bright red star	10 31 23	
B.A.C. 1391	10 44 54·5	

About three or four seconds before occultation of the three principal stars I noticed a diffraction phenomenon which I do not recollect to have previously observed, viz. that as the Moon approached each star the brilliancy of the latter completely obliterated the grey tint of the lunar surface at the point of contact, and a dark semicircle appeared thereupon up to the moment of disappearance, which in the case of each star was quite instantaneous.

The reappearance of θ^1 and θ^2 *Tauri*, so far as I could judge, was not so immediate.

I employed my Equatorial telescope of 6·8 inches aperture and 12 feet focal length, mag. power 144.

The Observatory, Crowborough, Sussex:
April 2, 1887.

Errata.

In General Tennant's paper, "The Orbit of Comet II., 1883," page 26, line 11,

for $L = 114^\circ\ 39'\ 01''.9$

read $l = 114\ 59\ 01.9$

In General Tennant's paper, "Notes on Reflecting Telescopes," page 258, line 8,

for $Ap = \frac{a}{16} = 2a + \frac{a}{16}\left\{24v^2 + 9v^4 + \frac{7}{2}v^6 + \&c.\right\}\frac{1024 + 640v^2 + 96v^4 + 4v^6 + v^8}{32 - 4v^2 - 3v^4}$

read $Ap = \frac{a}{16} \cdot \frac{1024 + 640v^2 + 96v^4 + 4v^6 + v^8}{32 - 4v^2 - 3v^4} = 2a + \frac{a}{16}\left\{24v^2 + 9v^4 + \frac{7}{2}v^6 + \&c.\right\}$